

GENERATION OF HIGH TEMPERATURE THERMAL RADIATION
BY LASER HEATING OF A CAVITY

Dissertation

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Fakultät für Physik
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vorgelegt von

Ricardo A. Pakula
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Abstract

The laboratory generation of very intense blackbody radiation with a temperature exceeding a million degrees seems possible with the help of modern pulsed laser and particle beams. The central idea is to focus the beam into a small cavity and to generate, via thermalization of its energy, intense thermal radiation in equilibrium with the heated walls. Important applications in science and technology can be envisaged for this scheme, for example the realization of a radiation standard in the soft x-ray region, scientific studies of radiation transport and thermodynamic properties of matter at high pressures and densities, and eventually inertial confinement fusion. In this thesis we attempt a first step towards its theoretical understanding and experimental realization using a pulsed laser beam.

The theoretical part of this work begins with a general discussion of the physics of radiation confinement. It is pointed out that the unavoidable losses of radiation connected with the heating of the wall determine the power requirements for the achievement of high temperatures inside the cavity. The calculation of the wall losses requires the solution of the time and space dependent hydrodynamic equations including radiative heat transfer. Using the concept of self-similarity, the partial differential equations are transformed into ordinary differential equations depending on a single dimensionless variable. It is shown that the temporal evolution of the wall heating can be described as a succession of four self-similar states, depending on the optical thickness and expansion of the heated material. The scaling laws for the hydrodynamic quantities, the range of validity and the border lines of the different regimes are obtained. One of the four states is of particular importance for the confinement of thermal radiation produced by a laboratory power source. This state, called the ablative heat wave and characterized by the simultaneous heating and expansion of the wall material, is solved and treated in detail for the first time in the approximation of radiative heat conduction.

As a result of the theoretical part of this thesis, detailed predictions are obtained for the conditions achievable in the cavity with an ideal power source, i.e. a source with optimal coupling to the radiation field in the

cavity. In particular a quality factor is introduced for the radiation confinement in analogy with a microwave cavity and calculated. This factor governs the approach to a Planckian spectrum and the degree of isotropy of the radiation. It becomes clear that even for favourable coupling only with the most powerful installations existing today a quality factor substantially larger than one can be achieved.

In the experimental part of this work, small hollow gold spheres of 300 μm and 1 mm diameter were heated with the high power laser system Asterix III, delivering about 100 Joules in 300 picoseconds. The laser beam was focussed into the entrance hole located at the front side of the cavity and the radiation field produced inside was observed through a small diagnostic hole on the back side of the sphere. The problem of premature closing of the entrance hole was investigated. It was established that for a satisfactory energy transmission into the cavity the entrance hole should be at least 150 μm in diameter. The centering of the laser beam on the entrance hole was achieved by positioning the cavities with the help of a telescope. In addition each shot was controlled by means of a x-ray pinhole camera.

Space resolved spectra of the radiation emitted by the cavity were obtained by means of a pinhole-grating spectrometer on film. The spectra were digitized with a computer supported densitometer and unfolded into absolute spectral energy density. Making use of the spatial resolution, it could be demonstrated that the analyzed radiation was radiated uniformly through the diagnostic hole. Brightness temperatures of 50 eV and 110 eV were measured for the large and small cavities, respectively.

A discussion of the specific problems of laser heating of a cavity preceeds the comparison between the experimental and theoretical results. It is assumed that the laser generates either a bath of thermal electrons, a source of plasma x-ray radiation or a source of suprathreshold electrons in the cavity; with the help of this model, the steps leading to the formation of a hot, optically thick wall layer emitting blackbody like thermal radiation are examined for each case applying general results of the theoretical part. With the possible exception of decoupling between the electron thermal bath and the wall, it is shown that a high degree of confinement and thermalization of the laser energy inside the cavity

should be achieved for the experimental conditions. This expectation is confirmed by the good agreement between the measured and predicted values of the radiated energy. In addition the spectral distribution of the emitted radiation approaches more closely the blackbody limit at the higher temperatures achieved in the smaller cavities as theoretically predicted.

It is concluded that the physical model of cavity heating, based on radiation confinement under approximate equilibrium conditions, is valid and its applicability improves further, if a more powerful source than used in the present experiment would be available.

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1. Introduction

The physics of radiation in thermal equilibrium with matter has been well known since Max Planck's discovery of the law of black-body radiation at the beginning of this century /1/. Until recently, however, it was not possible to generate black-body radiation with temperatures exceeding $10^4 - 10^5$ K in the laboratory. The difficulty encountered in generating high-temperature radiation fields is a consequence of the enormous increase of intensity with temperature. From the Stefan-Boltzmann law

$$S = \sigma T^4 ,$$

(where S is the radiant emittance, T the temperature in degree Kelvin and $\sigma = 5.67 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$ the Stefan-Boltzmann constant), one finds $S = 6 \times 10^{19} \text{ erg s}^{-1} \text{ cm}^{-2}$ for $T = 10^6$ °K, increasing with the fourth power of the temperature. The losses connected with such an enormous flux of energy are so high that the radiation field cannot be maintained by ordinary power sources.

The situation has changed in recent years, however, with the advent of modern pulsed power generators, such as pulsed laser and particle beams. These sources offer the possibility of generating radiation fields with temperatures of up to about 5×10^6 K in the laboratory. This possibility is of great scientific and technical interest, e.g. for the generation of an intense soft x-ray radiation source with a well-known spectral distribution, the laboratory study of phenomena of radiation hydrodynamics which otherwise occur only in stars, the investigation of matter at the extreme pressures generated and the production of energy by thermonuclear fusion.

The principle of generation of an intense radiation field by a laboratory source is shown in Fig. 1. The external source delivers energy into a cavity with a solid wall. With a laser source, a small hole for transmitting the beam is required; a particle beam could possibly penetrate a wall thin enough to transmit the beam but thick enough to contain the radiation field in the cavity. The rapid deposition of energy

in the cavity is supposed to heat the inner wall to a high temperature and to generate an intense black-body radiation field in equilibrium with the wall. For the applications envisaged it is important that at these elevated temperatures the radiative exchange of energy between different wall elements becomes so effective that it might be possible to establish very uniform conditions in the cavity even if the initial energy deposition by the source is not uniform. Furthermore, because photons propagate with the speed of light, the radiation field may be formed in the empty cavity before it fills with evaporated material from the wall.

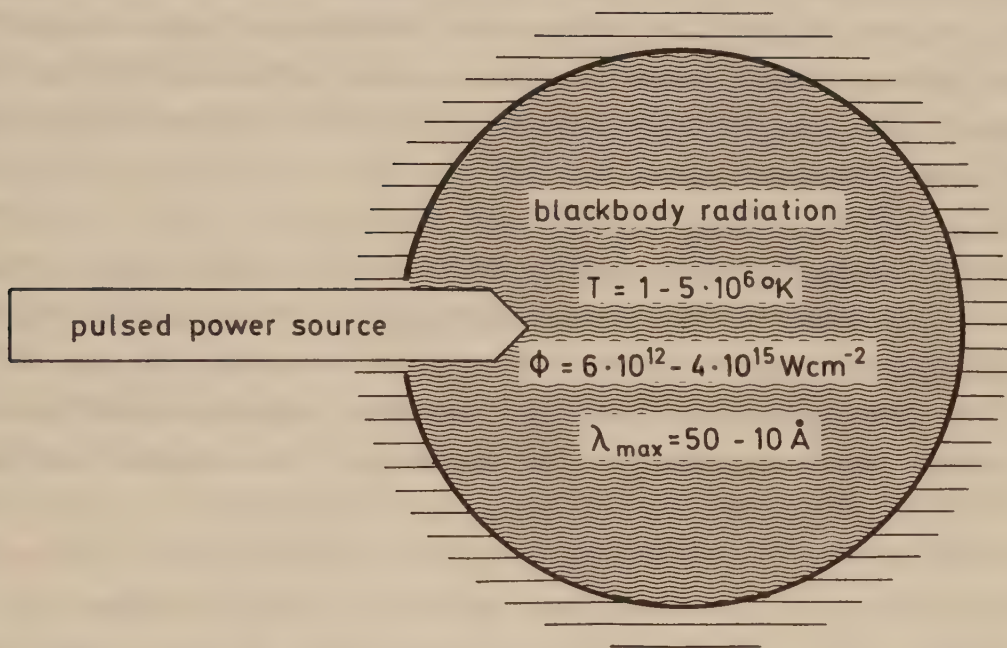


Fig. 1: Schematic representation of the generation of an isotropic black-body radiation field in a cavity by a laboratory pulsed power source.

The theoretical and experimental investigation of this scheme forms the subject of the present thesis. Particular attention is given to the laser, the pulsed power source used for the experimental work.

A crucial problem of this concept concerns the confinement of the radiation field by the cavity. Unlike for infrared or visible light, no

reflecting walls exists for soft X-rays, where the Planck distribution has its maximum for the temperatures under consideration. The incident photon flux is completely absorbed by the wall, and the radiation field can exist only in equilibrium with a hot layer on the inner surface of the cavity, re-emitting the incident radiation. The presence of wall material, which forces the photons to diffuse, thus reducing the outward energy flux and, in a sense, confining the radiation, has on the other hand the consequence that a fraction of the energy flux is used up for the heating of material. If the energy loss connected with the heating of wall material is too large, the desired intensity and uniformity may not be reached in the cavity. We note that the situation is not as favourable as in the interior of the sun, where the peripheral layers confine the intense radiation field in its center. Another difference to the static situation in the sun is that we deal here with a problem which is extremely time-dependent.

The theoretical part of this work is devoted to the confinement problem. In Sec. II we begin with a physical discussion of its essential features and establish the guideline for a theoretical treatment. In Sec. III we reconsider the classical problem of radiation hydrodynamics where a solid body is suddenly brought into contact with a radiation heat bath; this problem (which may be treated in planar geometry) is fundamental for the confinement of an intense radiation field by a wall. The concept of self-similarity /3-5/ is the usual method to attack problems of this type /6-13/. The difficulty one encounters here is that a self-similar solution (the famous Marshak wave /8/) is known only for the case where the density of the heated material is constant. However, for the conditions achievable with a laboratory source, expansion of the heated material occurs simultaneously with heating and must be taken into account. We shall show that a self-similar solution, which we call the ablative heat wave, exists also for this case.

In Sec. IV the frame of the theoretical treatment is widened to include not only the case of local thermodynamic equilibrium (LTE) but also nonequilibrium (NEQ) cases, that dominate in the initial heat-up phase of the cavity. It is shown that the heating of the wall can be treated as a sequence of self-similar regimes, which follow each other in time. The scaling laws for each regime are derived, the concept of border lines

introduced and a graphical discussion given in the T-t plane. The results are applied to cavities in Sec. V where also various questions like holes in the cavity, stored radiation energy, isotropy and spectral distribution are discussed. The main result of the theoretical part are then quantitative predictions for the heating of a cavity by a pulsed power source.

In the experimental part of this work (Sec. VI) heating experiments of small gold cavities (250-1000 μm diameter) with a pulse laser are described. The iodine laser Asterix III was used for these experiments, emitting either $100 \text{ J}/300 \text{ ps}/\lambda = 1.3 \text{ } \mu\text{m}$ or $30 \text{ J}/300 \text{ ps}/\lambda = 0.43 \text{ } \mu\text{m}$ pulses. The spectrum emitted through a small hole in the cavity is measured with a transmission grating spectrometer on film and thus the intensity of the radiation field in the cavity absolutely determined. The experimental results are analysed and the assumption of an ideal source examined for the laser case in Sec. VII. In particular the coupling of the laser beam with the wall via electron thermal conduction, plasma radiation and fast electrons is discussed. It is shown that a major fraction of the absorbed laser energy should indeed couple to the thermal radiation in the cavity. A comparison between the predicted and measured intensity and temperature of the radiation field concludes the thesis.

II. The physics of confinement of radiation in a cavity

As mentioned already, the theoretical part of this thesis is devoted to the confinement of an intense radiation field in the cavity. In this section we discuss the physical aspects of this problem in a more qualitative manner, but, at the same time, introduce the concepts used later for its quantitative treatment.

First we ask for the optimal choice of the wall material. As already mentioned, the scheme requires that the wall re-emits a large fraction of the incident energy back into the cavity. This can only occur if the mean free path of the radiation is smaller than the depth of the heated layer of the wall. Furthermore, at the temperatures considered, heating transforms the wall material into a highly ionized plasma. Let us assume for the moment that the absorption of radiation occurs in a fully ionized plasma in a three-body collision between ions with charge Z and number density N^+ , electrons with number density N^- , and photons (free-free and free-bound transitions are three-body processes). Noting [2] that the cross-section for emission (or absorption) of a photon in an electron-ion collision is proportional to Z^2 , we get the following proportionalities for the radiation mean free path ℓ_R :

$$\ell_R \sim (N^+ N^- Z^2)^{-1} \sim (N^{+2} Z^3)^{-1} \sim ((Z/m_i)^2 (N^+ m_i)^2 Z)^{-1} \sim (\rho^2 Z)^{-1}.$$

Here we have assumed quasi-neutrality $N^- = ZN^+$ and, as is appropriate for a fully ionized plasma, Z/m_i approximately constant (ρ is the density). The heated mass per unit area and likewise the internal energy deposited per unit area within an optical mean free path scale as $\rho \ell_R \sim 1/\rho Z$, i.e. an optically thick surface layer is generated with minimum expenditure of energy if a dense material with a large atomic number is chosen as wall material. This remains true also in the more realistic situation when the material is partially ionized and bound-bound transitions (two-body processes) contribute to absorption and reduce the scaling $\ell_R \sim \rho^{-\mu'}$ to an intermediate exponent $1 < \mu' < 2$.

The radiation (Rosseland) mean free path depends not only on the density, but also on the temperature. It is assumed in the following that it can be represented in the form of a power law (v is the specific volume, A_ℓ is a constant)

$$\ell_R = A_\ell T^j \nu^{\mu'} \quad (1)$$

This formulation is equivalent to a power law for the coefficient of radiative heat conduction of the form

$$\kappa = \kappa_0 T^v \nu^\mu, \quad (2)$$

where (see Ref. /2/, p. 654) $\kappa_0 = (16/3)\sigma A_\ell$, and $v = j+3$ and $\mu' = \mu$.

As a typical example of a high-Z material we use the values $j = 2$, $\mu' = 2$ for solid gold (see Appendix 2).

Let us now assume for the time being that the desired situation already exists in the cavity, i.e. the cavity is filled with an intense radiation field of temperature T . The inner surface of the cavity is heated up to the same temperature, radiation and matter are in local thermodynamic equilibrium (LTE). The diffusion of radiation into the wall has led to the formation of a nonlinear heat wave (see part III and IV and Ref. 2). Such a wave is characterized by a flat temperature plateau and a steep front owing to the strong (nonlinear) dependence of the coefficient of heat conductivity on the temperature. The situation is sketched in Fig.2.

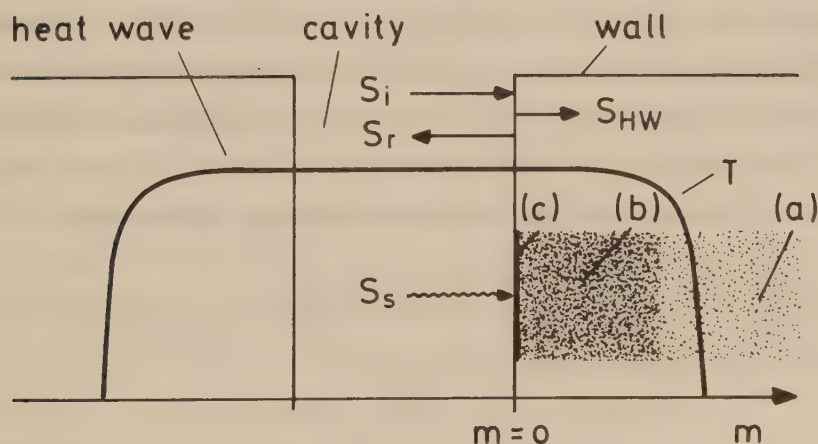


Fig. 2: Illustration of conditions at the inner wall of the cavity. S_i and S_r are the incident and reflected black-body emittance S_s the source intensity and S_{HW} the heat flux into the thermal wave. (a)-(c) are zones of energy deposition for different sources (see text). m is the Lagrangian mass coordinate; the density profile of the wall is shown for $t = 0$.

The role of the energy source is to maintain or even further enhance the temperature in the cavity. Whether a given source is suitable or not for this purpose obviously depends on the depth at which the source energy thermalizes. Three cases (a-c) are shown in Fig. 2.

In case (a) the source energy is deposited ahead of the heat wave front (of course the temperature distribution would be different from that shown in Fig. 2). This case is uneconomic because the source heats material which would not have been heated by diffusion of radiation from the interior of the cavity. The energy requirements for the heating of the cavity may be prohibitive for such a source.

Case (b) is a practical case, because the source deposits its energy in material which is heated by the heat wave in any case. However, an exact treatment of this case would be rather difficult because the details of the deposition mechanisms would have to be taken into account.

We therefore discuss the energy balance of the cavity with an ideal source (case (c)) for which the energy is assumed to thermalize in a (infinitely thin) surface layer of the wall. In this case the temperature has a maximum at the surface of the wall, with the consequence that the radiation field in the cavity has the highest possible temperature for a given source intensity. In other words, the energy required to heat the cavity to a given temperature is the lowest for this case and will be higher for any real source. However, the difference between the idealized case (c) and the practical case (b) is not expected to be very large. This is due to the very high radiative heat conductivity which is responsible for the plateau in the temperature profile of the heat wave and which tends to smooth out the details of energy deposition.

We now consider the energy balance of the surface layer of the wall, being given by

$$S_i - S_r + S_s - S_{HW} = 0. \quad (3)$$

Here S_s is the source strength (power per unit area), S_i and S_r the incident and re-emitted emittances of thermal radiation, respectively, and S_{HW} the flux supporting the heat wave. Note that S_{HW} is the net heat flux into the wall.

Let us assume now that the intensity distribution of the source and the temperature are completely uniform on the inner surface of the cavity. In this case no transfer of energy via thermal radiation occurs between different wall elements. Furthermore, it turns out (see Sec. V) that under conditions typical of heating with a laboratory source the radiation energy stored in the volume of the cavity may be neglected. This is a consequence of the small size of the cavities (≤ 1 cm) and the relatively modest temperatures ($\leq 5 \times 10^6$ K) which can be achieved. Also, compared with heating pulses of 10^{-10} s or longer, transit time effects through the cavity (a few tens of picoseconds at most) are negligible. In such a situation the net flux of radiation from the surface into the cavity is zero i.e. $S_i - S_r = 0$ and, consequently, $S_s = S_{HW}$ from eq. 3. The boundary condition for the heat wave is thus that the heat flux into the wave is equal to the (known) source intensity and we can interpret in this case the quantity S , defined as $S = S_i = S_r$ as the thermal radiation flux circulating inside the cavity.

In the closed geometry of a cavity the attainable temperature depends obviously on the insulating properties of the wall. In our case the interior of the cavity is "insulated" against the cold parts of the wall by the layer of hot material heated by the heat wave. Losses occur through the diffusion of radiation to the front of the heat wave. As the thickness of the wave increases with time, this diffusion process becomes more and more impeded by the presence of more hot material and the insulation of the radiation field improves. It is clear then that the temperature in the cavity must increase with time if a constant power is delivered from the source into the cavity.

A useful quantity for the following discussions is

$$N = S/S_{HW} = \frac{\text{thermal radiation flux in the cavity}}{\text{energy flux into the wall}}$$

It may be useful to point out that this quantity is defined in analogy to the Q-factor of a microwave cavity. We shall call it therefore in the following the quality factor. The essential difference is that in a microwave cavity a high Q-factor is achieved through reflection at the wall, whereas in our case the energy falling onto the wall is absorbed and reemitted as thermal radiation into the cavity.

The calculation of N requires the calculation of the thermal radiant emittance S in the cavity (as we have seen S_{HW} is equal to the known source intensity S_s) which is a very complicated problem in general. It becomes simple only in the limit where the cavity wall radiates as a blackbody; such a situation will be approached with time as the thickness of the heated layer and the temperature in the cavity increase. As the temperature rises, the emittance σT^4 of the blackbody radiation can, at least in principle, become much larger than the source intensity on the inner wall. This means that with time we can expect that in a uniformly irradiated, closed cavity

$$S \cong \sigma T^4 \gg S_s$$

or expressed in terms of N

$$N = S/S_{HW} \cong \sigma T^4/S_s \gg 1.$$

where T is the wall temperature. In this limit where we can speak of blackbody radiation in the cavity, N represents the ratio blackbody emittance/source intensity. The determination of N requires then the calculation of the wall temperature T for a given source intensity S_s .

Although we shall calculate N and other quantities of interest only for a uniformly heated cavity, the results obtained in this manner can be used to analyze situations where the heating of the cavity by the source is not uniform (at least if the deviations from uniformity are slight). As mentioned already, the possibility of distributing the source energy uniformly in the cavity by radiative transfer is of particular interest. It is obviously directly connected with the fact that the black-body intensity may become much larger than the source intensity with time. In this context it may be useful to point out that N allows still another interpretation, namely

$$N = \text{number of energy "reflections"}$$

in the cavity. This is directly seen from the relation $S = NS_s$ (obtained from the definition $N = S/S_{HW}$ with $S_{HW} = S_s$). Since the power source

(with intensity S_s) is the only source of energy in the cavity, this relation means that the black-body emittance S is generated by effectively N -fold "reflections" or re-emissions of the source energy before it is lost into the wall. From this point of view, with each re-emission the uniformity will increase, leading eventually to very uniform deposition as N becomes large.

We shall discuss the attainable uniformity in more detail in Sec. V. Here we should note, however, that in the limit $N \gg 1$ accurate values of N and other quantities of interest can be calculated with the assumption of uniform heating, because for $N \gg 1$ radiative smoothing becomes effective and uniform conditions are approached. This is the justification of the assumption of uniform heating of the wall by the source which is made throughout this paper.

Clearly the desired conditions in the cavity correspond to $N \gg 1$. It is very important to note that in this limit the diffusion of radiation into the wall may be calculated in the so-called approximation of radiation heat conduction. This considerably simplifies the mathematical treatment. In essence the problem of confinement of an intense radiation field then reduces to calculation of the wall temperature as a function of time for the boundary condition of a given heat flux into the wall. This requires solution of the time and space-dependent hydrodynamic equations including radiative heat conduction. A certain simplification is afforded by the fact that a planar solution is sufficient since we are interested in stages where the thickness of the heated (and possibly expanded) layer of material is still small compared with the radius of the cavity.

As is well known the penetration of heat into a wall occurs at such high temperatures in the form of a nonlinear heat wave. In particular, a very detailed discussion is given in Ref. /2/ for the case where the density of the wall material does not change upon heating. However, as we shall see, for typical conditions achievable with a laboratory source, hydrodynamic expansion of the heated material sets in during the heating pulse and must be taken into account. Expansion of the heated material lowers its mean density and hence, as we have discussed above, increases the amount of energy which has to be invested to form an optically thick layer. The additionally required energy is transformed

into kinetic energy of the expanding material, thus being lost for re-emission. We shall show in Sec. III that the heat wave with expansion, called the ablative heat wave, is also self-similar and calculate it in detail.

We emphasized above that the situation of real interest can be accurately treated in the conduction approximation, but this approximation is not valid initially, when the walls of the cavity are still relatively cold. In this early phase radiative energy transport is not yet important and the energy flux and the spatial deposition of energy depends very much on the characteristics of the source and its interaction with matter. Nevertheless, we will show in Sec. IV that it can also be treated in a general manner using the concept of self-similarity. These results will be used in the experimental section to discuss problems connected with the thermalization of a laser source. The expectation (confirmed by the analysis) is that for a powerful source this initial phase is passed so rapidly that it does not have much effect on the final conditions established in the cavity.

III. Self-similar expansion of dense matter due to heat transfer by nonlinear heat conduction

In this part we consider the problem of wall heating under the assumption that the condition $N \gg 1$ holds in the cavity and that radiation and matter are in local thermodynamical equilibrium.

III.1. General remarks about self-similar solutions

III.1.1 Statement of the problem

We begin our analysis by considering the following model: the one half-space is filled by a body of solid density which at time $t = 0$ is brought into contact with a thermal bath with a temperature of the order 10^5 to 10^8 °K (Fig. 3a). A steep temperature gradient is set up, leading to the diffusion of heat into the interior of the body. Simultaneously a high pressure is generated with the consequence that hydrodynamic expansion starts at the surface of the body. As time goes on the heat supplied by the bath is continuously converted into internal and kinetic energy of the heated material. Our interest is to calculate the net heat flux from the bath to the material as a function of time. Because heating and expansion occur simultaneously, the solution of that problem can only be found by solving the hydrodynamic equations in space and time, taking heat conduction into account.

This problem is in fact a classical problem of the physics of hydrodynamic phenomena /2/. For a constant bath temperature the heating of the body proceeds as follows: A supersonic heat wave propagates into the undisturbed material of solid density ρ_s (Fig. 3b). At the same time hydrodynamic expansion of the heated material into the vacuum begins and a compression wave travels into the solid. Since the speed of the heat wave is slowing down with time it is soon overtaken by the compression wave (Fig. 3c). The subsequent state where the heat wave is preceded by a shock wave and accompanied by hydrodynamic expansion, is maintained for later times. We call this state an ablative heat wave.

In our investigation we concentrate on the ablative heat wave with the aim of finding a self-similar solution of the asymptotic type i.e. a solution which the actual flow approaches in the limit $t \rightarrow \infty$.

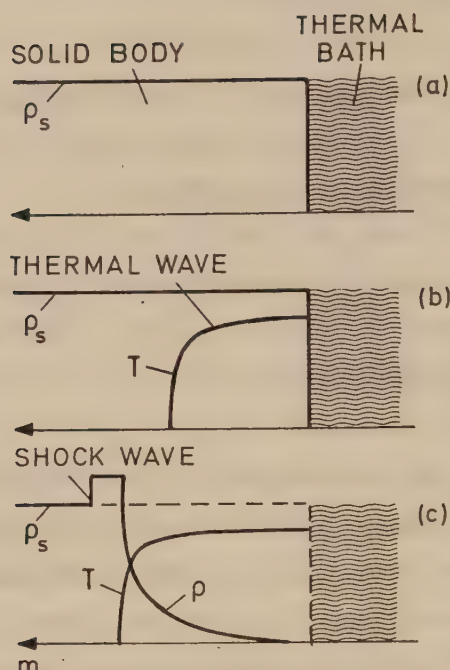


Fig. 3: Heating of a solid body in contact with a thermal bath.

- a) At time $t = 0$ the body is brought into contact with the thermal bath.
- b) For $t > 0$ first a nonlinear heat wave runs into the undisturbed material.
- c) Subsequently hydrodynamic motion of the heated material becomes important, the heat wave is overtaken by a shock wave and the ablative heat wave forms.

Our treatment will here be limited to the so-called conduction approximation. In this approximation the physical nature of the bath and its interaction with the material are left out of consideration; only the bath temperature appears as a boundary condition for the first Lagrangian cell of the body. As conduction mechanism in the material we deal mainly with radiation heat conduction in a multiply ionized material. It should be emphasized, however, that the analysis is performed for an arbitrary power dependence of the coefficient of heat conductivity on density and temperature and also applies to, in particular, the case of electronic heat conduction (see also Sect. VII). The range of validity of the conduction approximation will be discussed in section IV.

III.1.2 Hydrodynamic equations with nonlinear heat conduction

At the envisaged temperatures of the order of 10^5 to 10^8 °K the coefficient of heat conductivity κ depends on the temperature and density of the material. It is usually assumed to have a power dependence on these quantities. That is, with S the heat flux, T the temperature, and $v \equiv 1/\rho$ the specific volume (ρ the density) we have

$$S = -\kappa \frac{\partial T}{\partial x}; \quad \kappa = \kappa_0 T^\nu v^\mu.$$

The coefficient κ_0 and the exponents ν and μ depend on the conduction mechanism /2/. With radiation heat conduction one has $\nu \cong 4.5 - 6$, $\mu = 1.5 - 2$; with electron heat conduction in a fully ionized plasma $\nu = 5/2$, $\mu = 0$.

In planar geometry the hydrodynamic equations including nonlinear heat conduction, but not radiation pressure and energy, are /2/

$$\frac{\partial v}{\partial t} - \frac{\partial u}{\partial m} = 0, \tag{4}$$

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial m} = 0, \tag{5}$$

$$\begin{aligned} \frac{1}{\gamma - 1} \frac{\partial (pv)}{\partial t} + p \frac{\partial v}{\partial t} \\ = a \frac{\partial}{\partial m} \left((pv)^\lambda v^{\mu-1} \frac{\partial (pv)}{\partial m} \right) \left(= - \frac{\partial S}{\partial m} \right). \end{aligned} \tag{6}$$

Here m is the Lagrangian mass coordinate, u, v, p are, respectively, the velocity, specific volume and pressure, and γ the adiabatic index (assumed to be constant). To put the hydrodynamic equations into the form given above, we have taken the specific internal energy e as

$$e = pv/(\gamma - 1), \tag{7}$$

and the equation of state as

$$pv = KT^\delta. \tag{8}$$

The constant a is related to κ_0 , K , ν and δ by

$$a = \kappa_0 / (\delta K^{(\nu+1)/\delta}). \tag{9}$$

The exponent λ is related to ν and δ by

$$\lambda = (\nu + 1 - \delta) / \delta. \quad (10)$$

III.1.3 Self-similarity

In order to contrast clearly our results to previous ones, we find it necessary to start with a brief outline of the concept of self-similarity. On the basis of Buckingham's theorem /3/ it has been discussed, together with many applications, in the books of Sedov /4/ and Barenblatt /5/. We basically follow the notation of Barenblatt /5/.

Let a physical quantity g depend on n dimensional parameters $a_1, \dots, a_n$

$$g = g(a_1, \dots, a_k, a_{k+1}, \dots, a_n),$$

g is called a governed quantity, the parameters $a_1, \dots, a_n$ governing parameters. The latter have been grouped so that $a_1, \dots, a_k$ have independent dimensions (otherwise the choice of $a_1, \dots, a_k$ from $a_1, \dots, a_n$ is arbitrary). It is assumed that the dimensions of the parameters $a_{k+1}, \dots, a_n$ as well as of g are expressible in terms of power monomials of $a_1, \dots, a_k$. Then, according to the Buckingham theorem, g can be expressed as a product of a dimensional function g_1 and a dimensionless function $\tilde{G}$, where $\tilde{G}$ depends on $n-k$ dimensionless similarity parameters $\xi_1, \dots, \xi_{n-k}$

$$g = g_1(a_1, \dots, a_k) \tilde{G}(\xi_1, \dots, \xi_{n-k}),$$

g_1 is of the form

$$g_1(a_1, \dots, a_k) = a_1^{\alpha_1} \dots a_k^{\alpha_k}.$$

The dimensionless similarity parameters are

$$\xi_1 = a_{k+1} \cdot a_1^{\alpha_1} \dots a_k^{\alpha_k}, \dots, \quad \xi_{n-k} = a_n \cdot a_1^{\alpha_1 - \alpha_k} \dots a_k^{\alpha_k - \alpha_k}.$$

Suppose now that $n-k=1$, i.e. that the number of dimensional parameters of the problem exceeds the number of independent dimensions by one. There then exists just one similarity parameter ξ , and we have

$$g = g_1 \tilde{G}(\xi).$$

In this case we call the problem self-similar. Introducing the flow variables in this form reduces the hydrodynamic equations from a system of partial differential equations to a set of ordinary differential equations.

Let us now consider our problem. We choose a system of units of measurement with $k=3$ independent units of mass $[M]$, length $[L]$ and time $[\theta]$. The hydrodynamic equations depend on three dimensional governing parameters, namely m with $[m] = ML^{-2}$, t with $[t] = \theta$, and a with $[a] = M^{\mu+1} L^{-2\lambda-3\mu-1} \theta^{2\lambda-1}$. Additional parameters come from the boundary conditions.

As a boundary condition at the vacuum-material interface we specify the specific internal energy in the first Lagrangian cell of the body. This is entirely equivalent to a boundary condition for the temperature through eqs. (7) and (8); the internal energy is preferred only because we formulate the problem in mechanical units. We give the boundary condition for a modified internal energy $W = (\gamma-1)e$, allowing for a time-dependence, in the form

$$\begin{aligned} (pv)|_{m=0} &= (\gamma-1)e_v = W_v = W_v^* t^\tau; \\ [W_v^*] &= L^2 \theta^{-(2+\tau)}. \end{aligned} \tag{11}$$

The subscript v denotes the vacuum-material interface. This boundary condition adds another dimensional parameter W_v^* (note that its dimension depends on τ). We then have $n = 4$ dimensional parameters $a_1 = t$, $a_2 = a$, $a_3 = W_v^*$, $a_4 = m$. Because $n-k = 1$ we can form only a single dimensionless variable ξ_1 , given by

$$\xi_1 = m a^{\alpha_1} t^{\tau_1} W_v^{*\omega_1}. \tag{12}$$

If the problem contained no other dimensional parameter it would obviously be self-similar.

Unfortunately the problem of heating of a solid body contains at least another dimensional parameter, namely the initial density ρ_s of the body. This parameter enters the problem through the boundary conditions at the interface with the solid. As long as it is important, the problem is not self-similar.

III.1.4 Asymptotic self-similar solution

The observation that in the ablative heat wave regime the initial density loses its importance as a governing parameter of the problem as time goes to infinity is of central importance for the present investigation. In this limit an asymptotic self-similar solution exists for the general case of a time-varying boundary temperature.

Let us first present a physical argument in support of this observation. The heat supplied at the vacuum boundary has to diffuse through an increasing mass of hot, expanding plasma in order to heat new layers of cold material in the interior part of the body. Thus, if the boundary temperature is kept constant, the rate of plasma production will slow down with time. On the other hand, the characteristic expansion velocity of the plasma, namely the sound velocity corresponding to the boundary temperature, remains constant. Hence, by virtue of mass conservation, the characteristic plasma density must decrease with time and will therefore become much less than ρ_s . One might therefore expect that, as time goes to infinity, the hydrodynamic flow of the conduction heated material becomes independent of the numerical value of ρ_s .

In order to elaborate this argument further, let us form the characteristic plasma density ρ_1 by dimensional analysis

$$\rho_1(t, a, W_v^*) = \left(\frac{t^{1-\tau(\lambda-1)}}{a W_v^{*\lambda-1}} \right)^{-1/(\mu+1)} \quad (13)$$

As we shall see later (see sect. III.4.2) this density corresponds, within a factor of order unity, to the density at the sonic point. We are interested in the heating (rather than cooling) of the body i.e. in the case $\tau \geq 0$. If the time exponent of ρ_1 is inspected it is found that $\rho_1 \rightarrow 0$ for $t \rightarrow \infty$ in the range $0 \leq \tau < \tau_M \equiv 1/(\lambda-1)$. The characteristic plasma density becomes with time much smaller than the solid density not only for a constant boundary temperature, but also for a boundary temperature that increases with time proportional to t^τ in this range of τ .

In order to assess the role of the parameter ρ_s in the limit $t \rightarrow \infty$ we consider the plasma-solid interface. As shown in Fig. 3c the conduction

heated plasma is connected with the undisturbed solid through a shock wave. The shock wave takes up the whole recoil momentum of the expanding plasma but only a fraction of the order $(\rho_1/\rho_s)^{1/2}$ of the convective energy flux of the plasma. If $\rho_1 \rightarrow 0$ for $t \rightarrow \infty$ the ablation of material occurs eventually like from a rigid wall of infinite density which takes up momentum, but no energy; the heat flux supplied at the vacuum boundary is then entirely transformed into convective and internal energy of the plasma.

In this limit the hydrodynamic flow in the conduction zone does no longer depends on the properties of the shock wave region and may be calculated by simplified boundary conditions at the plasma-solid interface, corresponding to infinite density, zero temperature and zero velocity (the pressure is of course finite, but can be determined only through the solution of the problem). Most important, these boundary conditions do not contain any dimensional parameter, in particular no longer the parameter ρ_s . The asymptotic ablative heat wave therefore depends indeed only on the four parameters t, a, W_v^* and m and is self-similar.

The self-similar solution obtained in this way will approach the exact solution of the problem everywhere in the heated material except for a thin layer near the ablation front where the density in the similarity solution exceeds solid density. However, as long as one is not specially interested in the details of this transition layer, it can be ignored because its fractional mass and energy content tend to zero with time.

III.1.5 The equivalence of different boundary conditions

In the preceeding section we considered the heating of a solid body for the case of a given time-dependent boundary temperature. Indeed we will solve the problem in this form in the main part of this chapter.

However, sometimes the problem may be posed in a different manner. For example, one might be interested in a self-similar solution where the heat flux into the plasma is given as a boundary condition. Other examples of

possible interest are solutions where the pressure exerted onto the solid is specified or where the characteristic plasma density remains at a fixed value independent of time. Each of these solutions would be characterized by a dimensional parameter, called Q in the following, which is different in each case. For the example mentioned, Q would be a heat flux S_0 , a pressure p_0 , and a density ρ_0 , respectively. Here the subscript 0 is used to distinguish these parameters from the corresponding variables.

We shall show now that characterization of the problem by such a parameter Q is equivalent to a time-dependent boundary temperature with a certain time exponent τ . Physically this means that a condition imposed onto the solution may be met if the boundary temperature is suitable "programmed".

The equivalent τ for a given Q can be found as follows. Formally the parameter Q (Q plays the role of an additional parameter a_5) introduces a second dimensionless similarity parameter

$$\xi_2 = Q a^{\alpha_2} t^{\tau_2} W_v^{*\omega_2} \quad (14)$$

into the problem. τ is now determined in such a manner that the problem remains self-similar. This is possible if, through the proper choice of τ , ξ_2 is made a time-independent constant rather than a variable. In this case, because ξ_2 does not depend on m and t , the partial derivatives $\partial \xi_2 / \partial m$ and $\partial \xi_2 / \partial t$ disappear and hence the dimensionless hydrodynamic equations remain unchanged (compare Sect. III.3.1) and are, in particular, a function of the single variables ξ_1 (the constant ξ_2 whose value depends on the numerical value of Q , appears as a parameter in the functions $\tilde{G}$ only). With the dimensions of Q taken as $[Q] = M^{q_1} L^{q_2} \theta^{q_3}$, with the dimensions of W_v^* given by eq. (11), and with $\tau_2 = 0$ this condition, applied to eq. (14), yields readily the desired equivalence between τ and Q through dimensional analysis:

$$\tau = \frac{-2q_1(3\mu + 2) - 2q_2(\mu + 1) - 2q_3(\mu + 1)}{q_1(2\lambda + 3\mu + 1) + q_2(\mu + 1)}. \quad (15)$$

We shall solve in this paper the ablative heat wave problem for four discrete values of τ , equivalent to the dimensional parameters W_0 (resp. T_0), S_0 , p_0 , and ρ_0 (see sect. III.4). The corresponding values of τ from eq. (15) may be found in Tables I, II and III.

The equivalence of τ and Q is of interest also with respect to previous work on the subject (see sct. III.1.6). A problem that has one governing parameter too much can obviously be made self-similar by imposing a certain time-dependence on another parameter of the problem (we chose the boundary temperature). Thus the benefit of self-similarity is achieved by sacrificing the free choice of the time dependence of another governing parameter of the problem. In this case only a special self-similar solution is obtained which in general will not correspond to the boundary conditions of interest.

III.1.6 Previous work

As outlined above, self-similarity is achieved in our problem if one of the five (including ρ_s) governing parameters is discarded. Various approaches have been taken in this respect.

The simplest method of obtaining a self-similar solution of our set of fundamental equations is to eliminate time; in this way a stationary solution is obtained. In fact, approximate stationary solutions taking expansion into account have been given for the case of laser heating /6/ and heating by thermal radiation /7/. However, these solutions do not apply to a truly planar case, which is necessarily non-stationary and miss important aspects of the problem connected with the time dependence of the physical quantities.

Perhaps the most widely known self-similar solution is the thermal wave in a medium of constant density. Because the density in this case does not depend on m and t it is readily seen from the energy equation that the problem only depends on a single parameter $a\rho_s^{1-\mu}$ and not on a and ρ_s separately. Hence the number of dimensional parameters is reduced by one and the problem is self-similar. This solution is described in detail in /2/ and /8/. It is not applicable to the ablative heat wave where expansion is important and the density is not constant.

The possibility that a special self-similar solution of the ablative heat wave problem may be obtained if the solid density is kept as a governing

parameter was apparently pointed out first by Marshak /8/. He noticed that for the case of radiation heat conduction in a fully ionized plasma the problem is self-similar if the boundary temperature varies as $T_v \sim t^{1/5}$. A similar approach was taken by Anisimov /9/ in the treatment of laser heating of a solid target where electronic heat conduction is dominant. Here the temporal variation of the boundary temperature required for self-similarity is $T_v \sim t^{2/3}$. An extensive treatment of the laser heating problem based on this type of self-similar solutions was recently given by Barrero and Sanmartin /10,11,12/.

Both time exponents are readily obtained from our considerations in sect. III.1.5. With $Q = \rho_s$, i.e. $[Q] = [\rho_s] = ML^{-3}$ and $q_1 = 1$, $q_2 = -3$, $q_3 = 0$, one obtains $\tau = 1/(\lambda-1)$ from eq. (15). Here $\tau = \tau_M$ that means that $Q = \rho_s$ is the limiting case discussed in Sect. III.1.4 where the characteristic plasma density becomes independent of time. For radiation heat conduction in a fully ionized plasma with $\delta = 1$ and $\nu = 6$ one obtains $\lambda = (\nu+1-\delta)/\delta = 6$ and $\tau = 1/5$ (Marshak's case). For electronic heat conduction in a fully ionized plasma ($\delta = 1$, $\nu = 5/2$) one gets $\tau = 2/3$ (Anisimov's case).

A common feature in all of these approaches is that the solid density ρ_s is kept as a governing parameter of the problem. This leads to the restrictions imposed onto the self-similar solutions mentioned above, like stationary flow, the absence of expansion or a prescribed temporal variation of the boundary condition. However, as we have discussed in Sect. III.1.4, ρ_s eliminates itself with time and is not really an essential parameter of the ablative heat wave problem. The previous treatments /8-12/ appear therefore unnecessarily restrictive.

To our knowledge Afanas'ev et al. /13/ used for the first time the assumption of an infinitely dense body to simplify the somewhat related problem of laser heating of a solid target in the optically thin regime. This idea is exploited here in a systematic manner for the problem of the heating of a body by nonlinear conduction. The observation that the ablative heat wave is asymptotically self-similar offers us the possibility to obtain self-similar solutions for a whole range ($0 \leq \tau < \tau_M$) of the time exponent τ (actually the scaling laws for the various hydrodynamic quantities can be obtained immediately from dimensional analysis, even if the problem is not solved exactly). In addition the problem is conceptually simplified to a great extent.

III.2 Dimensional analysis

We now proceed with standard dimensional analysis as briefly described in sct. III.1.3. The dimensions of the quantities which appear in the hydrodynamic equations and in the boundary condition are $[t] = \theta$, $[a] = M^{\mu+1} L^{-(2\lambda+3\mu+1)} \theta^{2\lambda-1}$, $[W_v^*] = L^2 \theta^{-(2+\tau)}$, $[m] = ML^{-2}$, $[\gamma] = 1$, $[v] = M^{-1} L^3$, $[p] = ML^{-1} \theta^{-2}$, $[u] = L \theta^{-1}$.

With $a_1 = t$, $a_2 = a$, $a_3 = W_v^*$, $a_4 = m$ we form a single dimensionless similarity parameter

$$\xi = m t^{\tau_\xi} a^{\alpha_\xi} W_v^{*\omega_\xi}. \quad (16)$$

We locate the plasma vacuum interface at $\xi = 0$, and the plasma-solid interface at $\xi = \xi_f$. The mass m_f between these boundaries is given by

$$m_f = \xi_f / (t^{\tau_\xi} a^{\alpha_\xi} W_v^{*\omega_\xi}),$$

hence the expression for ξ may be written as

$$\xi = \xi_f m / m_f. \quad (17)$$

This means that ξ and m are proportional to each other with a time-dependent proportionality factor. The variables u, v, p are written in the form

$$\begin{aligned} u(m, t) &= t^{\tau_u} a^{\alpha_u} W_v^{*\omega_u} \tilde{U}(\xi) = u_1(t) \tilde{U}(\xi), \\ v(m, t) &= t^{\tau_v} a^{\alpha_v} W_v^{*\omega_v} \tilde{V}(\xi) = v_1(t) \tilde{V}(\xi), \\ p(m, t) &= t^{\tau_p} a^{\alpha_p} W_v^{*\omega_p} \tilde{P}(\xi) = p_1(t) \tilde{P}(\xi). \end{aligned} \quad (18)$$

For use in the subsequent parts of this work we give explicitly the expressions for the temperature T , the heat flux S and the total energy E_{tot} written in the same manner:

$$\begin{aligned} T(m, t) &= T_1(t) \tilde{T}(\xi); & \begin{cases} T_1(t) = (p_1 v_1 / K)^{1/\delta}, \\ \tilde{T}(\xi) = (\tilde{P} \tilde{V})^{1/\delta} \end{cases} \\ S(m, t) &= S_1(t) \tilde{S}(\xi); & \begin{cases} S_1(t) = a (p_1 v_1)^{\lambda+1} v_1^{\mu-1} \xi_f / m_f, \\ \tilde{S}(\xi) = (\tilde{P} \tilde{V})^{\lambda} \tilde{V}^{\mu-1} \frac{\partial(\tilde{P} \tilde{V})}{\partial \xi} \end{cases} \end{aligned}$$

$$E_{\text{tot}}(t) = E_1(t) \tilde{E}(\xi_f); \quad \begin{cases} E_1(t) = u_1^2 m_f / \xi_f, \\ \tilde{E}(\xi_f) = \int_0^{\xi_f} \left(\frac{\tilde{P}\tilde{V}}{\gamma - 1} + \frac{\tilde{U}^2}{2} \right) d\xi. \end{cases}$$

The expressions for $E_{\text{tot}}(t)$ are derived in Appendix 1. There it is also shown that the ratio of the internal and kinetic energies is a time-independent constant in this problem.

We note that the values of the quantities at the boundary of the plasma with the vacuum and solid (i.e. at the front of the wave) are denoted by the subscripts v and f, respectively. For example, the temperature T_v at the vacuum boundary is given by

$$T_v \equiv T_1(t) \tilde{T}(\xi = 0) = T_1(t).$$

Here we have used the normalization

$$\tilde{T}(\xi = 0) = \tilde{T}_v = 1,$$

which is applied throughout this work. The pressure p_f at the interface with the solid is

$$p_f = p_1(t) \tilde{P}(\xi = \xi_f) = p_1(t) \tilde{P}_f.$$

The exponents in eqs. (16) and (18) are now determined by dimensional analysis. One finds for the similarity parameter

$$\xi = m / (a W_v^{*\lambda + (\mu - 1)/2} t^{\mu + \tau[\lambda + (\mu - 1)/2]} 1/(\mu + 1),$$

for the dimensional part of the variables u, v , and p

$$u_1(t) = W_v^{*1/2} t^{\tau/2},$$

$$v_1(t) = (a^{-1} W_v^{*1 - \lambda} t^{1 - \tau(\lambda - 1)})^{1/(\mu + 1)},$$

$$p_1(t) = (a W_v^{*\lambda + \mu} t^{\tau(\lambda + \mu) - 1})^{1/(\mu + 1)},$$

and for the dimensional part of the variables T, S , and E_{tot}

$$T_1(t) = T_v = (W_v^{*} t^{\tau} K^{-1})^{1/\delta},$$

$$S_1(t) = (a W_v^{*\lambda + (3\mu + 1)/2} t^{\tau[\lambda + (3\mu + 1)/2] - 1})^{1/(\mu + 1)},$$

$$E_1(t) = (a W_v^{*\lambda + (3\mu + 1)/2} t^{\tau[\lambda + (3\mu + 1)/2]} 1/(\mu + 1).$$

The exponents τ_ξ, τ_v, τ_p , and τ_u are given by

$$\tau_\xi = [-\mu - \tau(\lambda + \mu/2 - 1/2)]/(\mu + 1),$$

$$\tau_v = [1 - \tau(\lambda - 1)]/(\mu + 1),$$

$$\tau_p = [\tau(\lambda + \mu) - 1]/(\mu + 1),$$

$$\tau_u = \tau/2.$$

III.3 Integration of the hydrodynamic equations

III.3.1 The hydrodynamic equations in dimensionless form

If the dependent variables are written in the form $g(m, t) = g_1(t)\tilde{G}(\xi)$ and since

$$\frac{\partial g}{\partial m} = g_1 \frac{d\tilde{G}}{d\xi} \frac{\partial \xi}{\partial m}, \quad \frac{\partial g}{\partial t} = \frac{dg_1}{dt} \tilde{G} + g_1 \frac{d\tilde{G}}{d\xi} \frac{\partial \xi}{\partial t},$$

the hydrodynamic equations reduce from a system of partial differential equations to a system of ordinary differential equations:

$$-\tau_v \tilde{V} - \tau_\xi \xi \frac{d\tilde{V}}{d\xi} + \frac{d\tilde{U}}{d\xi} = 0, \quad (19)$$

$$\tau_u \tilde{U} + \tau_\xi \xi \frac{d\tilde{U}}{d\xi} + \frac{d\tilde{P}}{d\xi} = 0, \quad (20)$$

$$\begin{aligned} & \left(\frac{\gamma\tau_v + \tau_p}{\gamma - 1} \right) \tilde{P}\tilde{V} + \frac{\tau_\xi}{\gamma - 1} \xi \frac{d(\tilde{P}\tilde{V})}{d\xi} + \tau_\xi \xi \tilde{P} \frac{d\tilde{V}}{d\xi} \\ & = \frac{d}{d\xi} \left((\tilde{P}\tilde{V})^\lambda \tilde{V}^{\mu-1} \frac{d(\tilde{P}\tilde{V})}{d\xi} \right). \end{aligned} \quad (21)$$

As boundary conditions we have at the plasma-vacuum boundary $\xi = 0$, $\tilde{V} = +\infty$, $\tilde{P}\tilde{V} = 1$, at the plasma-solid boundary $\xi = \xi_f$, $\tilde{U} = 0$, $\tilde{V} = 0$, $\tilde{P} = \tilde{P}_f$. Note that ξ_f and $\tilde{P}_f$ are a priori unknown.

Mathematically, eqs. (19-21) are equivalent to a first-order system of five equations if $\tilde{U}, \tilde{V}, \tilde{P}$, $d\tilde{P}/d\xi$ and $d\tilde{V}/d\xi$ are considered as independent variables. A complete set of boundary conditions therefore consists

of initial values for all of these variables at a common ξ . Obviously, the physical boundary conditions noted above are incomplete and do not allow integration to be started, neither at the plasma-vacuum nor at the plasma-solid boundary. This difficulty can be overcome with the help of approximate solutions.

III.3.2 Approximate solutions

a) Approximate solution at the plasma-solid boundary

We try to find an analytical approximation for $\xi \rightarrow \xi_f$ of the form

$$\tilde{P}(\xi) = \tilde{P}_f; \quad \tilde{V}(\xi) = \tilde{V}_0(\xi_f - \xi)^\beta.$$

Making use of the boundary conditions, we find to a first-order approximation in $(\xi_f - \xi)$

$$\tilde{U} = -\tau_\xi \xi_f \tilde{V}_0 (\xi_f - \xi)^{1/(\lambda + \mu - 1)}, \quad (22)$$

$$\tilde{V} = \tilde{V}_0 (\xi_f - \xi)^{1/(\lambda + \mu - 1)}, \quad (23)$$

$$\tilde{P} = \tilde{P}_f - \tau_\xi^2 \xi_f^2 \tilde{V}_0 (\xi_f - \xi)^{1/(\lambda + \mu - 1)}, \quad (24)$$

where

$$\tilde{V}_0 = [-\gamma(\gamma - 1)^{-1}(\lambda + \mu - 1)\tau_\xi \tilde{P}_f^{-\lambda} \xi_f]^{1/(\lambda + \mu - 1)}.$$

It is interesting to note that we get for the exponent of $(\xi_f - \xi)$ the same value as Marshak /8/ in his dimensionless solution for the problem with constant pressure.

b) Approximate solution at the plasma-vacuum boundary

For the case of a constant boundary temperature ($\tau = 0$) an approximate solution can also be given near the plasma-vacuum boundary. We look for solutions of the form

$$\tilde{V}(\xi) = \tilde{V}_0 \xi^{\beta_1}; \quad \beta_1 < 0,$$

with

$$\tilde{P}(\xi)\tilde{V}(\xi) = \tilde{W}_v,$$

and

$$(\tilde{P}\tilde{V})^\lambda \tilde{V}^{\mu-1} \frac{d(\tilde{P}\tilde{V})}{d\xi} = \tilde{S}_v.$$

Matching coefficients and exponents of ξ at both sides of equations (19-21), we obtain in a third-order approximation

$$\begin{aligned} \tilde{U} &= \tilde{U}_0 + (\tau_v - \tau_\xi)\tilde{V}_1 \ln \xi + [\tau_v + (\mu - 1)\tau_\xi] \tilde{V}_2 \mu^{-1} \xi^\mu \\ &\quad + (\tau_v + \mu\tau_\xi)(\mu + 1)^{-1} \tilde{V}_3 \xi^{\mu+1}, \\ \tilde{V} &= \tilde{V}_1 \xi^{-1} + \tilde{V}_2 \xi^{\mu-1} + \tilde{V}_3 \xi^\mu, \\ \tilde{P} &= \tau_\xi(\tau_\xi - \tau_v)\tilde{V}_1 \xi - \tau_\xi [\tau_v + (\mu - 1)\tau_\xi] (\mu + 1)^{-1} \tilde{V}_2 \xi^{\mu+1} \\ &\quad - \tau_\xi(\tau_v + \mu\tau_\xi)(\mu + 2)^{-1} \tilde{V}_3 \xi^{\mu+2}, \end{aligned}$$

where

$$\begin{aligned} \tilde{V}_1 &= [\tilde{W}_v \tau_\xi^{-1} (\tau_\xi - \tau_v)^{-1}]^{1/2}, \\ \tilde{V}_2 &= \tilde{S}_v (\mu + 1) \tilde{V}_1^{-(2\lambda + \mu)} \mu^{-2} \tau_\xi^{-(\lambda + 1)} \\ &\quad \times (\tau_\xi - \tau_v)^{-\lambda} (2\tau_\xi - \tau_v)^{-1}, \\ \tilde{V}_3 &= \frac{[(\gamma\tau_v + \tau_p)/(\gamma - 1) - \tau_\xi] (\mu + 2) [\tau_\xi(\tau_\xi - \tau_v)]^{\mu/2}}{\tilde{W}_v^{\lambda + \mu/2 - 1} (\mu + 1) \tau_\xi [2\tau_\xi - (\mu + 3)\tau_v]}. \end{aligned}$$

If $\tilde{W}_v$ and $\tilde{S}_v$ are specified, the approximate solution is fully determined.

III.3.3 Numerical integration

For numerical integration eqs. (19-21) were written as a first-order system of five equations and numerically integrated on a desk-top computer by means of a standard Runge-Kutta routine.

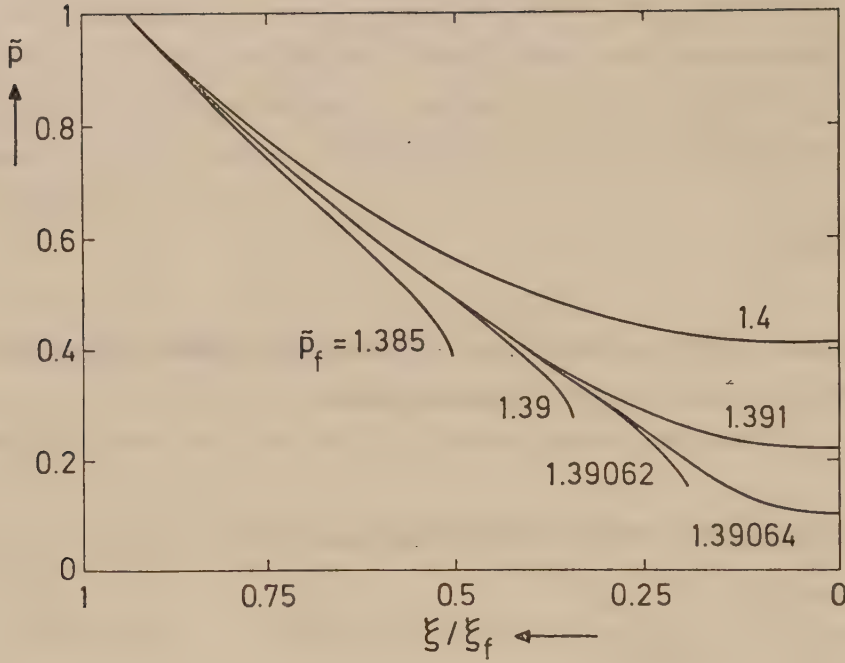


Fig. 4: Determination of the pressure at the solid-plasma boundary by numerical iteration.

Integration was started near the plasma-solid boundary with a set of initial values calculated from the approximate solutions (22-24). Because the scale factor ξ_f in eqs. (22-24) is unknown, a preliminary coordinate ξ^* was chosen by setting $\xi_f^* = 1$. The second open parameter in eqs. (22-24), the dimensionless pressure at the plasma-solid boundary $\tilde{p}_f$ (all quantities in the ξ^* frame are denoted by small characters, e.g. $\tilde{p}(\xi^*)$ etc.), was then determined numerically by successive approximations from the conditions that the solution has to fulfill the proper boundary conditions at the plasma-vacuum boundary. The procedure is illustrated in Fig. 4 for $\tilde{p}$, which has to fulfill the boundary condition $\tilde{p}_v = 0$. Once $\tilde{p}_f$ has been determined in this manner, numerical integration gives the solutions in ξ^* space, including the values at the boundaries.

With the arbitrary choice of the ξ^* scale, one obtains, in general, a value $\tilde{w}_v \neq 1$ instead of the desired boundary condition $\tilde{w}_v = 1$ (i.e. $W = W_v$ at the vacuum boundary). This difficulty can be overcome by rescaling the solution with the help of a similarity transformation.

As may be verified by insertion, the set of differential equations (19-21) admits a similarity transformation, i.e. it remains unchanged for a

transformation $\xi_1 \rightarrow \xi$ of the form $\xi = c\xi_1$, provided the variables are also transformed according to

$$\begin{aligned}\tilde{V}(\xi) &= \tilde{v}(\xi_1)c^{\beta_1}, \\ \tilde{U}(\xi) &= \tilde{u}(\xi_1)c^{\beta_1+1}, \\ \tilde{P}(\xi) &= \tilde{p}(\xi_1)c^{\beta_1+2},\end{aligned}\tag{25}$$

where c is an arbitrary constant and β_1 is given by

$$\beta_1 = 2(1 - \lambda)(2\lambda + \mu - 1)^{-1}.$$

To rescale the numerical results from the ξ^* to the ξ frame, the constant c is determined from the condition

$$\tilde{p}_v \tilde{v}_v c^{2(\beta_1+1)} = \tilde{P}_v \tilde{V}_v = 1,$$

i.e. from the numerical value of the product $\tilde{p}_v \tilde{v}_v$ determined by the integration in ξ^* space. The results of the numerical integration will be presented already in terms of the final variables $\tilde{U}(\xi)$, $\tilde{V}(\xi)$, and $\tilde{P}(\xi)$.

III.3.4 Transformation of the solutions into real space

Once the problem has been solved in dimensionless variables, the results have to be transformed back into real space. We take the origin of the spatial x -coordinate at the plasma-solid boundary, with the positive x -direction towards the vacuum. With the definition of the Lagrangian mass coordinate $dm = -v^{-1}dx$ and eq. (17), one has

$$dx = -(m_f/\xi_f)v_1(t)\tilde{V}(\xi)d\xi.$$

Insertion of the expressions for m_f and ξ_f and integration yield

$$x = -\frac{m_f}{\xi_f}v_1(t)\int_{\xi_f}^{\xi}\tilde{V}(\xi')d\xi' = -\frac{W_v^{1/2}t}{\xi_f}\int_{\xi_f}^{\xi}\tilde{V}(\xi')d\xi'.$$

This relation may be rewritten by introducing a variable

$$y \equiv xW_v^{-1/2}t^{-1}\xi_f = -\int_{\xi_f}^{\xi}\tilde{V}(\xi')d\xi'.\tag{26}$$

We call y the similarity coordinate in real space; it is related to ξ through an integral over the solution $\tilde{V}(\xi)$.

III.4 Results

III.4.1 Dimensional dependences

As already outlined in the introduction, we apply in this investigation a boundary condition of the form $W_V = (\gamma-1)e_V = W_V^* t^\tau$. The exponent τ is related through eq. (15) to a quantity Q which is a temporally constant governing parameter of the problem. In Table I we listed the dimensional part of the hydrodynamic variables for several choices of τ and hence Q . τ was chosen such that one obtains for Q either the specific internal energy W_0 (or the temperature T_0 through the relation $W_0 = KT_0^\delta$), heat flux S_0 , pressure p_0 , or density ρ_0 .

The numerical examples calculated explicitly are for a wall of solid gold. In this case (see Appendix 2) one has $\lambda=3$, $\mu=2$. Table II gives the dimensional dependences for this case. It is seen, for example, that a constant temperature can be maintained by a decreasing flux $S_1 \sim t^{-1/3}$,

TABLE I. General dimensional relations. Here λ and μ are related to j and μ' , the exponents of the Rosseland mean free path $l_R \propto T^j v^{\mu'}$, by $\lambda = (j + 4 - \delta)/\delta$ and $\mu = \mu'$ [see Eq. 10 and Appendix 2]. Here δ is the temperature exponent in the equation of state $p v = K T^\delta$ [Eq. (8)]. Note that W_0 and T_0 are related by $W_0 = K T_0^\delta$.

Q	τ	ξ	u_1	v_1	p_1
W_0	0	$m/(a W_0^{\lambda + (\mu-1)/2} t^\mu)^{1/(\mu+1)}$	$W_0^{1/2}$	$(a^{-1} W_0^{1-\lambda} t)^{1/(\mu+1)}$	$(a W_0^{\lambda + \mu} t^{-1})^{1/(\mu+1)}$
S_0	$2/(2\lambda + 3\mu + 1)$	$m/(a^2 S_0^{2\lambda + \mu - 1} t^{2\lambda + 3\mu - 1})^{1/(2\lambda + 3\mu + 1)}$	$(a^{-1} S_0^{\mu + 1} t)^{1/(2\lambda + 3\mu + 1)}$	$(a^{-3} S_0^{2-2\lambda} t^3)^{1/(2\lambda + 3\mu + 1)}$	$(a S_0^{2(\lambda + \mu)} t^{-1})^{1/(2\lambda + 3\mu + 1)}$
p_0	$1/(\lambda + \mu)$	$m/(a p_0^{2\lambda + \mu + 1} t^{2\lambda + 2\mu - 1})^{1/(2\lambda + 2\mu)}$	$(a^{-1} p_0^{\mu + 1} t)^{1/(2\lambda + \mu)}$	$(a^{-1} p_0^{1-\lambda} t)^{1/(\lambda + \mu)}$	p_0
ρ_0	$1/(\lambda - 1)$	$m/(a^{-1} \rho_0^{2\lambda + \mu - 1} t^{2\lambda - 1})^{1/(2\lambda - 1)}$	$(a^{-1} \rho_0^{\mu + 1} t)^{1/(2\lambda - 1)}$	ρ_0^{-1}	$(a^{-1} \rho_0^{\lambda + \mu} t)^{1/(\lambda - 1)}$

TABLE II. Dimensional relations for our standard case $\delta = 3/2$ and $(j, \mu') = (2, 2)$, corresponding to $(\lambda, \mu) = (3, 2)$ (multiply ionized gold, see Appendix 2).

Q	τ	ξ	u_1	v_1	p_1	E_1	S_1
W_0	0	$m/(a W_0^{7/2} t^2)^{1/3}$	$W_0^{1/2}$	$(a^{-1} W_0^{-2} t)^{1/3}$	$(a W_0^3 t^{-1})^{1/3}$	$(a W_0^{13/2} t^2)^{1/3}$	$(a W_0^{3/2} t^{-1})^{1/3}$
S_0	$2/13$	$m/(a^2 S_0^7 t^{11})^{1/13}$	$(a^{-1} S_0^3 t)^{1/13}$	$(a^{-3} S_0^{-4} t^3)^{1/13}$	$(a S_0^{10} t^{-1})^{1/13}$	$S_0 t$	S_0
p_0	$1/5$	$m/(a p_0^9 t^9)^{1/10}$	$(a^{-1} p_0^3 t)^{1/10}$	$(a^{-1} p_0^{-5} t)^{1/10}$	p_0	$(a^{-1} p_0^{13} t^{11})^{1/10}$	$(a^{-1} p_0^{13} t)^{1/10}$
ρ_0	$1/2$	$m/(a^{-1} \rho_0^7 t^5)^{1/4}$	$(a^{-1} \rho_0^3 t)^{1/4}$	ρ_0^{-1}	$(a^{-1} \rho_0^5 t)^{1/2}$	$(a^{-3} \rho_0^{13} t^7)^{1/4}$	$(a^{-3} \rho_0^{13} t^3)^{1/4}$

a constant pressure requires only a slightly increasing flux $S_1 \sim t^{1/10}$, whereas a constant plasma density can only be maintained by a flux increasing according to $S_1 \sim t^{3/4}$.

We note that the quantities with the subscript o should be understood as characteristic parameters of the problem and the relations given in Tables I and II as scaling relations. The expressions still lag a numerical factor which depends on how the parameter Q enters through the boundary conditions and which can be determined only by integration of the hydrodynamic equations. In order to obtain, for example, in the case $Q = S_o$ exact expressions as a function of the heat flux S_v at the vacuum boundary one has to replace S_o with $S_o = S_v / \tilde{S}_v$; this yields the numerical factor provided the boundary value $\tilde{S}_v$ of the dimensionless function $\tilde{S}$ has been determined. Some boundary values of the dimensionless functions will be given in Table III (see Sect.III.4.2).

III.4.2 Numerical results

Let us discuss in some detail the numerical results obtained for the case of a constant boundary temperature ($\tau = 0$), $j = 2$, $\mu' = 2$ and $\gamma = 3/2$. The variation of the dimensionless hydrodynamic variables is shown in Fig. 5.

The dimensionless temperature $\tilde{T}$ and the dimensionless heat flux $\tilde{S}$ steadily decrease from the vacuum boundary to the plasma-solid boundary, where they become zero. At the vacuum boundary one has $\tilde{T}_v = 1$ (as specified by the boundary condition); the dimensionless heat flux there has the value $\tilde{S}_v = 1.68$. The dimensionless pressure $\tilde{P}$ is zero at the vacuum boundary and attains a finite value $\tilde{P}_f = 0.758$ at the plasma-solid boundary (with the coordinate $\xi_f = 0.65$). The dimensionless flow velocity $\tilde{U}$ is zero at the solid and diverges at the vacuum boundary. The dimensionless density $\tilde{R}$ becomes infinite at the solid and zero at the vacuum boundary (with the corresponding behaviour for $\tilde{V}$).

The sonic point is located at a fixed value of ξ determined by the condition

$$M = \frac{u}{C_u} = \frac{W_v^{1/2} \tilde{U}(\xi)}{W_v^{1/2} \tilde{W}(\xi)^{1/2}} = \frac{\tilde{U}(\xi)}{\tilde{W}^{1/2}(\xi)} = 1.$$

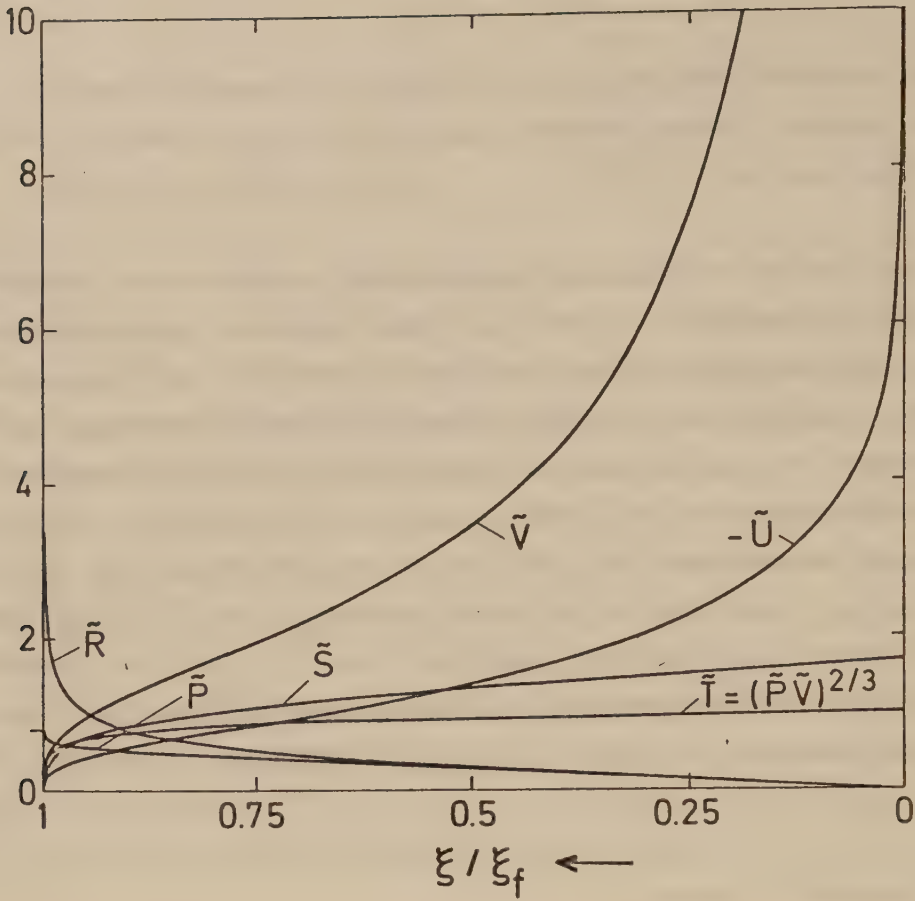


Fig. 5: Spatial variation of the dimensionless hydrodynamic variables in the conduction heated plasma. The solid-plasma interface is at $\xi/\xi_f = 1$, the plasma vacuum interface at $\xi/\xi_f = 0$.

Its numerical value is $\xi_{M=1} = 0.47$. The density at the sonic point is

$$\rho_{M=1} = \rho_1(t) \tilde{R}(\xi_{M=1}) = 0.48 \rho_1(t).$$

This relation illustrates the physical meaning of the characteristic plasma density $\rho_1(t)$. It is, within a factor of order unity, equal to the density at the sonic point.

The degree to which the approximate solutions approach the numerical ones may be judged from Fig. 6.

The curve y versus ξ , which afford transformation of the results into real space (eq. 26), is shown in Fig. 7. Figure 8a shows the curves of Fig. 5 in y -space. From these graphs the values of the gasdynamic quantities can be evaluated for a given x and t .

Q	j	μ'	τ	γ	ξ_f	$(\tilde{P}\tilde{V})_v$	$\tilde{S}_v$	$\tilde{P}_f$	$\xi_{M=1}$	$\tilde{R}_{M=1}$
T_o	2	1	0	5/4	.36	1	.93	.29	.24	.24
	1	1	0		.42		1.01	.32	.26	.27
	1	2	0		.56		1.89	.62	.39	.42
	2	2	0		.56		2.03	.65	.41	.42
S_o	2	1	1/5	5/4	.28	1	1.22	.35	.18	.24
	1	1	3/13		.33		1.33	.37	.19	.28
	1	2	6/35		.47		2.11	.63	.32	.42
	2	2	2/13		.48		2.39	.70	.34	.42
P_o	2	1	1/4	5/4	.27	1	1.29	.36	.17	.25
	1	1	3/10		.31		1.42	.39	.18	.29
	1	2	3/13		.44		2.17	.63	.30	.42
	2	2	1/5		.46		2.49	.71	.32	.43
ρ_o	2	1	1/2	5/4	.22	1	1.56	.42	.14	.26
	1	1	3/4		.27		1.81	.47	.16	.32
	1	2	3/4		.35		2.54	.69	.23	.44
	2	2	1/2		.37		2.96	.78	.26	.45
T_o	2	2	0	3/2	.65	1	1.68	.76	.47	.48

TABLE III. Calculated values for four similarity solutions in each of which a different parameter Q is held constant in time [Q is either a temperature T_o (note $W_o = KT_o^\delta$) a heat flux S_o , a pressure p_o , or a density ρ_o]. Here τ is the time exponent of the boundary temperature required to achieve a temporally constant Q , γ the adiabatic index. The values are given for four different pairs of exponents (j, μ') of the Rosseland mean free path $l_R \sim T^j v^{\mu'}$. Here ξ_f is the front coordinate of the wave (plasma-solid interface), the dimensionless temperature is normalized such that $(\tilde{P}\tilde{V})_v = T_o^\delta = 1$. The boundary values for the dimensionless heat flux $\tilde{S}_v$ (the index v denotes the plasma-vacuum boundary), the dimensionless pressure $\tilde{P}_f$ at the wave front, and in addition, the coordinate of the sonic point $\xi_{M=1}$ and the dimensionless density $\tilde{R}_{M=1}$ at the sonic point are given. Here $\delta = 3/2$ always.

In this representation it is also readily seen that the flow far from the solid approaches an isothermal rarefaction (linear increase of $\tilde{U}$ with y), as one might expect for a constant boundary temperature.

Figure 8b shows, in addition, the dimensionless specific kinetic energy $\tilde{E}_k$, the dimensionless specific internal energy $\tilde{E}_i$, and, in addition to $\tilde{U}$, the dimensionless sound velocity $\tilde{C}_u = (\tilde{P} \cdot \tilde{V})^{1/2}$. The sonic point is located at $y = 0.26$.

The other cases given in Table II, corresponding to the given parameters S_o, P_o, ρ_o , were also integrated numerically. The shape of the calculated curves is, however, qualitatively very similar to the case with a constant boundary temperature; they are therefore not reproduced here. Instead we give only the boundary values and the coordinates of the plasma-solid boundary and of the sonic point in Table III.

Fig. 6: Comparison of the analytical approximate solutions (dashed lines) with the numerical ones (solid lines). One of the dashed lines corresponds to the approximation at the vacuum boundary whereas the other to the solid body boundary.

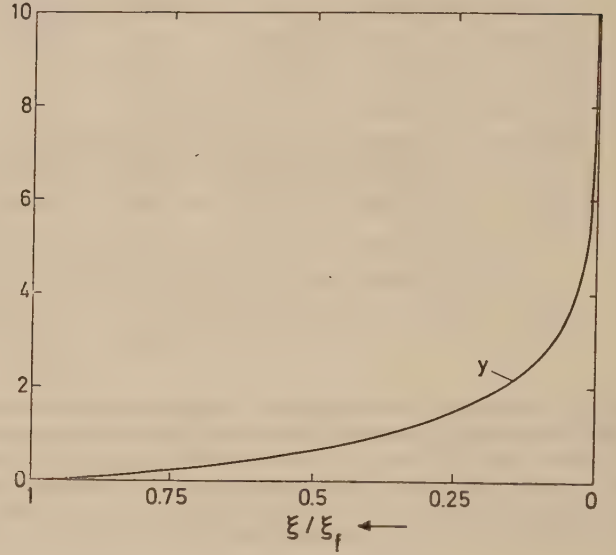
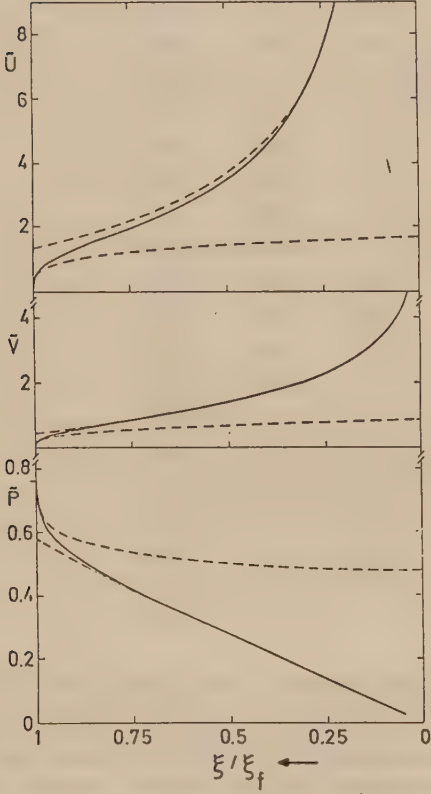


Fig. 7: The dimensionless variables y (the similarity variable in real space) versus ξ/ξ_f .

The calculations for Table III were performed for our standard values $(\lambda, \mu) = (3, 2)$ and, because the exponents λ and μ depend on the physical model for three other pairs of exponents as well. We remember that λ and μ [compare eqs. (1),(2) and App. 2] are related to j and μ' , the exponents of the Rosseland mean free path $l_R \sim T^j v^{\mu'}$, by $\lambda = (j+4-\delta)/\delta$ and $\mu = \mu'$ [see eq. (10) and Appendix 2]. Here δ is the temperature exponent in the equation of state $p v = K T^\delta$ [eq. (8)], and was chosen as $\delta = 3/2$. Besides the standard case $(j, \mu') = (2, 2)$, the pairs $(j, \mu') = (2, 1)$, $(1, 2)$, and $(1, 1)$ were chosen. At low temperature and high density, the case $(2, 2)$ will tend to the case $(1, 1)$, the latter corresponding to the maximum opacity possible (see Bernstein and Dyson Maximum Opacity Theorem /14/). The exponents $(2, 1)$ and $(1, 2)$ are intermediate.

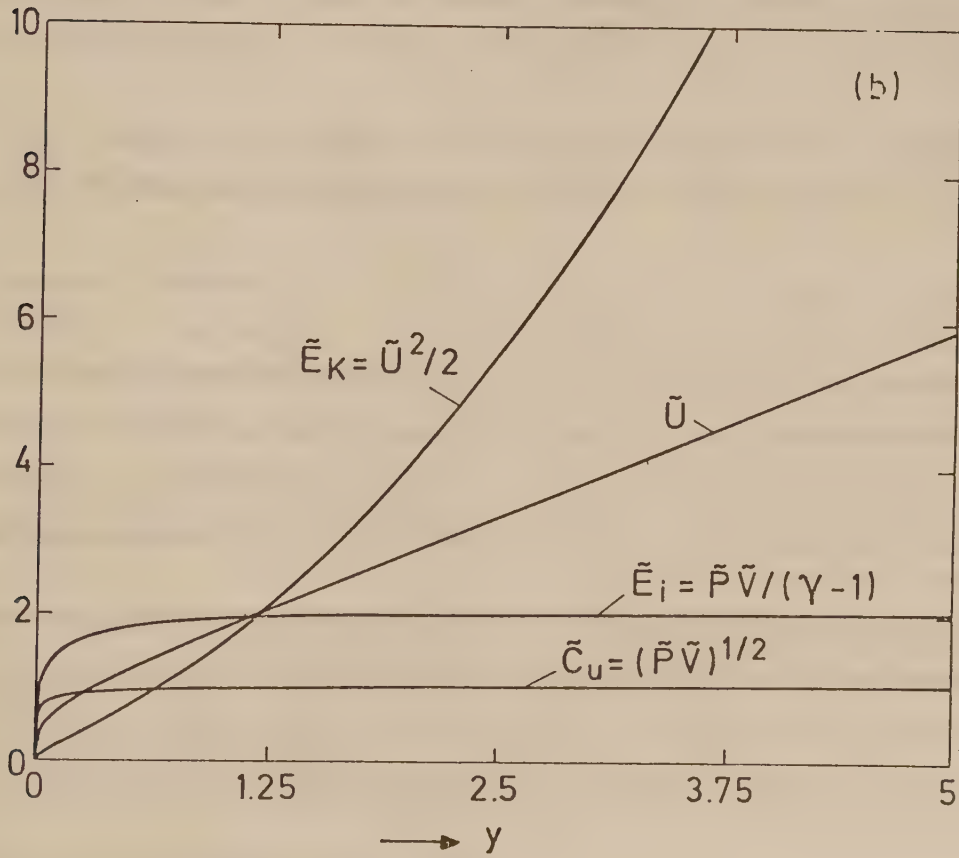
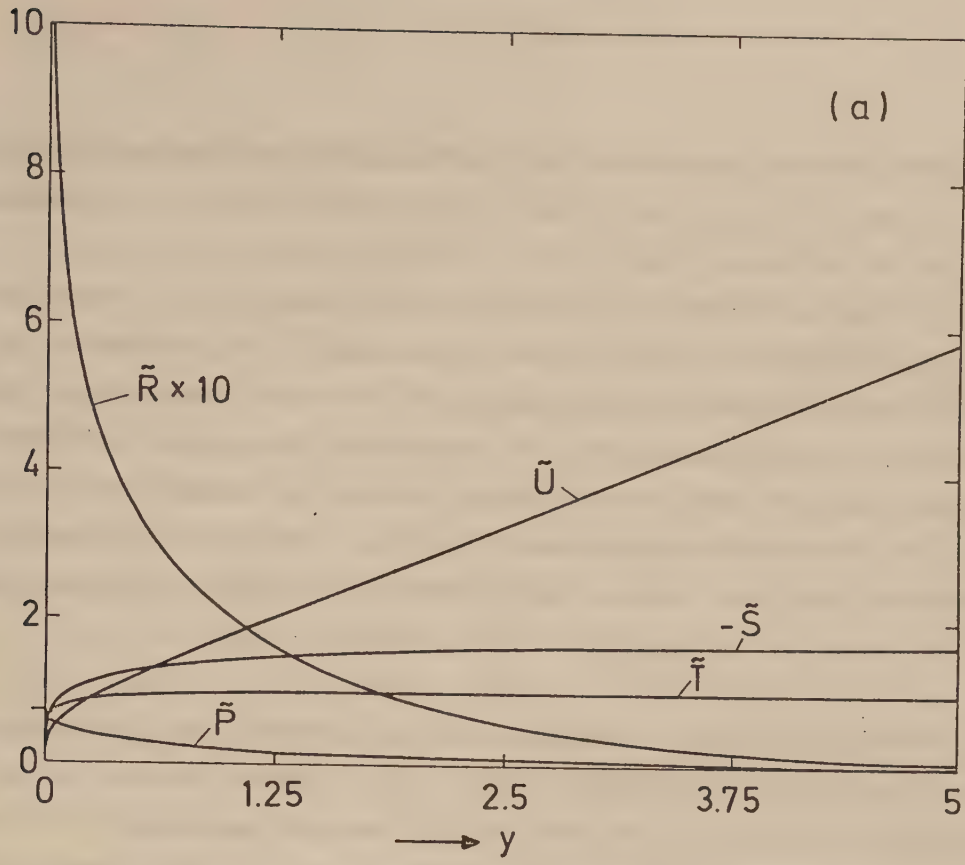


Fig. 8a: Graphs of the numerical solution for the dimensionless density $\tilde{R}$, expansion velocity $\tilde{U}$, energy flux $\tilde{S}$, Temperature $\tilde{T}$ and pressure $\tilde{P}$ as a function of the dimensionless distance y .

b: Graphs of the numerical solution for the dimensionless kinetic energy $\tilde{E}_K$, internal energy $\tilde{E}_i$, expansion velocity $\tilde{U}$ and sound velocity $\tilde{C}_u$ as a function of the dimensionless distance y .

The case of constant density $Q = \rho_0$ corresponds to Marshak's solution. Here $\xi_2 = \rho_s/\rho_1(t) = \rho_s/\rho_0$ is a time-independent constant. The boundary condition of an infinitely dense body is appropriate in the limit $\rho_0 \ll \rho_s$ or $\xi_2 \gg 1$; this corresponds to the limiting case $\alpha \ll 1$ calculated by Sanmartin and Barrero /10-12/ (their parameter α is related to ξ_2 by $\xi_2 \sim \alpha^{-3/2}$). From a mathematical point of view their calculations appear very similar to ours (neglecting the fact that they deal with electron heat conduction and take the critical density of laser radiation into account); the main difference lies in the fact that in the present work the restriction on the time-variation of the boundary temperature is removed.

Because Marshak's solution has found some attention in the literature, it may be useful to complement Table II by Tables IV and V for the Marshak case. Both tables are for a fully ionized plasma ($\delta = 1$), Table IV for radiation heat conduction ($\nu = 6, \mu = 2$), Table V for electron heat conduction ($\nu = 5/2, \mu = 0$).

This completes our specific analysis of the ablative heat wave. Its range of validity will be obtained from the more general treatment in the next section.

TABLE IV. Dimensional relations for $Q = \rho_0$ (Marshak's case) for radiation heat conduction in a fully ionized plasma ($\delta = 1$), and $(j, \mu') = (3, 2)$ as assumed by Marshak.

ξ	u_1	v_1	p_1	E_1	s_1	T_1
$m/(aW_0^{11/2}t^{16/3})^{1/3}$	$W_0^{1/2}t^{1/8}$	$(aW_0^3)^{-1/3}$	$(aW_0^8t^{3/5})^{1/3}$	$(aW_0^{19/2}t^{39/10})^{1/3}$	$(aW_0^{19/2}t^{19/10})^{1/3}$	$W_0K^{-1}t^{1/5}$

TABLE V. Dimensional relations for the case $Q = \rho_0$ for electronic heat conduction in a fully ionized plasma ($\delta = 1$). For electronic heat conduction, $(\nu, \mu) = (5/2, 0)$; ν and μ are the exponents of thermal conductivity $k \propto T^\nu \nu^\mu$. This case was treated by Anisimov.

ξ	u_1	v_1	p_1	E_1	S_1	T_1
$m/(aW_0^2t^{4/3})$	$W_0^{1/2}t^{1/3}$	$a^{-1}W_0^{-3/2}$	$aW_0^{5/2}t^{2/3}$	$aW_0^3t^2$	aW_0^3t	$W_0K^{-1}t^{2/3}$

IV. General approach to the problem of wall heating using self-similar solutions

In this section we analyze the problem of wall heating in a more general manner. Besides the ablative heat wave and the heat wave at constant density, both valid in the LTE approximation, we take into account also self-similar solutions of the nonequilibrium (NEQ) type where the energy transfer is dominated by source radiation. We shall show that in this way an approximate treatment of the influence of source properties on the heating of the cavity is possible; the results are used to obtain the limits of the LTE approximation and will be applied later in Sec. VII to the laser-heating of a cavity. For a fictitious source a complete description of wall heating is obtained and graphically discussed. The results are obtained in planar geometry i.e. they apply to a cavity where the expansion of heated wall material is much smaller than the cavity radius. This is not a serious restriction, however, because this situation corresponds to the case of interest.

IV.1 Four types of self-similar solutions

Our general treatment of the wall problem will be based on dimensional analysis and the concept of self-similarity (as briefly discussed in Sec. III). We shall represent the time evolution of the wall heating by a family of self-similar solutions. The emphasis is on the identification and discussion of physically different regimes rather than on numerical accuracy. The self-similar solutions, i.e. the profiles of density, temperature etc. are not explicitly calculated; they may be found in the literature cited. We shall only deal with so-called characteristic or mean quantities which are formed by dimensional analysis from the governing parameters of the problem. Nevertheless the accuracy of this treatment should not be underestimated; normally the boundary values of interest (e.g. the heat flux at the vacuum-plasma boundary) only deviate from the characteristic quantity by a factor of order unity.

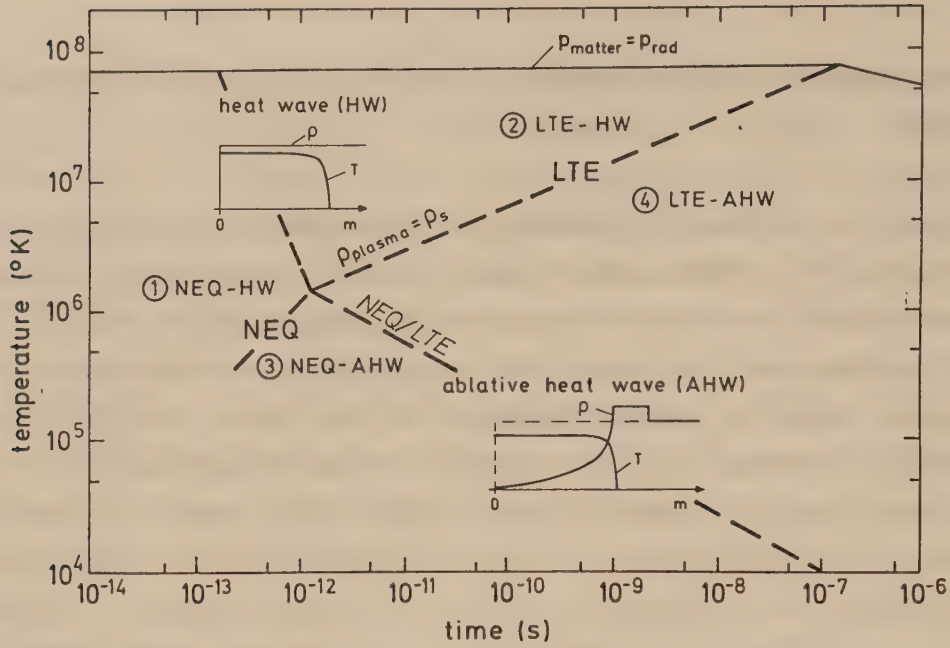


Fig. 9: The four regimes of self-similar flow with their border lines in the T - t plane (T is the matter temperature). Radiation pressure becomes important above the limiting curve $p_{\text{matter}} = p_{\text{rad}}$. The insets show schematically the density and temperature profiles in the heat wave and ablative heat wave regime.

In the conduction approximation the source term in the energy equation (6) becomes

$$-\partial S / \partial m = (\partial / \partial m) (\kappa \partial T / \partial x).$$

With the expressions $\kappa = \kappa_0 T^{\nu} \mu$ and $W = K T^{\delta}$ one obtains

$$-\partial S / \partial m = a (\partial / \partial m) (W^{\lambda} \mu^{-1} \partial W / \partial m),$$

with $a = \kappa_0 / (\delta K^{(v+1)/\delta})$ and $\lambda = (v+1-\delta)/\delta$ as in eqs. (9) and (10).

For the initial phase we assume that the heating of the material is dominated by energy deposition from the source; the exchange of energy between adjacent fluid elements by conduction is neglected. The source term in the energy equation is taken as

$$-\partial S / \partial m = \nu \partial S / \partial x = S \nu \ell_s^{-1},$$

where ℓ_s is the mean free path of photons or particles from the source. We assume that the expression for ℓ_s can be written as

$$\ell_s = \ell_0 W^s v^r, \quad (27)$$

i.e. ℓ_s depends on the internal energy and specific volume of the heated material in the form of a power law.

Our analysis will take into account four physically different regimes for which self-similar solutions of this set of equations exist. A brief characterization of these regimes follows (typical profiles of T and ρ are shown by the insets of Fig. 9):

i) Regime 1: Heat wave in a medium of constant density driven by non-equilibrium radiation (= NEQ-HW). The nonequilibrium radiation may be photon or particle radiation directly from the source or secondary radiation produced by interaction of source radiation with matter. An equilibrium radiation field at the temperature of the matter may also exist, but the temperature is not yet high enough for the energy flux associated with it to become comparable with the flux delivered by the source. Because the radiation mean free path of the source radiation is temperature-dependent, a heat wave develops, but the heated matter does not yet expand. This type of wave has been treated by Nemchinov /15/.

ii) Regime 2: Heat wave in a medium of constant density where matter and radiation are in local thermodynamic equilibrium (= LTE-HW). The heated layer is optically thick and the approximation of radiation heat conduction is applicable. This type of heat wave has been treated by Marshak /8/ and is extensively discussed by Zeldovich and Raizer /2/.

iii) Regime 3: Ablative heat wave driven by nonequilibrium radiation (= NEQ-AHW). This regime differs from regime 1 by the fact that the heated material expands during heating. This type of wave has been treated in the context of laser heating of a solid substance by Afanasiev et al. /13/.

iv) Regime 4: Ablative heat wave with matter and radiation in local thermodynamic equilibrium (= LTE-AHW). This regime differs from regime 2 in that the material expands during heating. A self-similar solution for this regime was given in part III. This solution is of the asymptotic type, i.e. approached by the flow in the limit $t \rightarrow \infty$ (in fact, if the boundary conditions do not vary extremely rapidly in time, each of the regimes 1 - 3 terminates in regime 4). Computer simulations have shown that, despite its asymptotic character, the wave is already an excellent approach to the actual flow at the beginning of its estimated range of validity /16/.

IV.2 Governing parameters

The governing parameters of our problem are m, t , a parameter A_i for each regime i , obtained from the energy equation, and another parameter from the boundary condition (for the definition of governing parameters see Sect. III.1.3 and Ref. /5/). In order to determine A_i , we write down the energy equation for each regime:

$$(1/(\gamma-1))\partial W/\partial t = S/\ell_0 W_s^s v_s^{r-1} \quad \text{NEQ-HW,} \quad (28)$$

$$(1/(\gamma-1))\partial W/\partial t = (a v_s^{\mu-1})(\partial/\partial m)(W^\lambda \partial W/\partial m) \quad \text{LTE-HW,} \quad (29)$$

$$(1/(\gamma-1))\partial W/\partial t + p \partial v/\partial t = S/\ell_0 W_s^s v_s^{r-1} \quad \text{NEQ-AHW,} \quad (30)$$

$$(1/(\gamma-1))\partial W/\partial t + p \partial v/\partial t = a(\partial/\partial m)(W^\lambda v_s^{\mu-1} \partial W/\partial m) \quad \text{LTE-AHW.} \quad (31)$$

where $v_s \equiv 1/\rho_s$

It is seen that for the four regimes the governing parameters A_i are

$$A_1 = \ell_0 v_s^{r-1}; \quad [A_1] = M \theta^{2s} L^{-2(s+1)} \quad \text{NEQ-HW,}$$

$$A_2 = a v_s^{\mu-1}; \quad [A_2] = M^2 \theta^{2\lambda-1} L^{-2(\lambda+2)} \quad \text{LTE-HW,}$$

$$A_3 = \ell_0; \quad [A_3] = M^r \theta^{2s} L^{-2s-3r+1} \quad \text{NEQ-AHW,}$$

$$A_4 = a; \quad [A_4] = M^{\mu+1} \theta^{2\lambda-1} L^{-(2\lambda+3\mu+1)} \quad \text{LTE-AHW.}$$

IV.3 Dimensional analysis

We use the cgs system of mechanical units with 3 independent basic units: [length] = L, [time] = θ and [mass] = M.

Applying well-known methods and results of dimensional analysis /4,5/, we find that each case is separately self-similar with a dimensionless similarity variable $\xi_i(m, t, A_i, Q)$ where the functional dependence on m, t, A_i , and Q is in the form of a power monomial. We give here ξ_i explicitly only for the case $Q = W_0$ with $[W_0] = L^2/\theta^2$. The reason for this choice is simply that in the following we shall derive for other choices of Q (namely S_0 , p_0 , and N_0) relations of the form $W_0 = W_0(Q, A_i, t)$; eliminating W_0 from the expressions for ξ_i given below one obtains readily for these cases $\xi_i(m, t, A_i, Q)$ (of course ξ_i can be also found directly through dimensional analysis for any Q).

For the case $Q = W_0$ one finds by dimensional analysis:

$$\begin{aligned}
 \xi_1 &= m/(A_1 W_0^S) \equiv m/m_1 && \text{NEQ-HW,} \\
 \xi_2 &= m/(A_2 W_0^\lambda t)^{1/2} \equiv m/m_2 && \text{LTE-HW,} \\
 \xi_3 &= m/(A_3 W_0^{(r+2s-1)/2} t^{r-1})^{1/r} \equiv m/m_3 && \text{NEQ-AHW,} \\
 \xi_4 &= m/(A_4 W_0^{\lambda+(\mu-1)/2} t^\mu)^{1/(\mu+1)} \equiv m/m_4 && \text{LTE-AHW.}
 \end{aligned} \tag{32}$$

A special characteristic of the nonlinear waves considered here is that the waves are of finite extent. The wave amplitude is zero for $\xi_i \geq \xi_{fi}$, where ξ_{fi} characterizes the wave front and is numerically of order one.

From (32) we can find the heated mass:

$$m_{fi} = m_i \xi_{fi} \quad (i = 1, 2, 3, 4).$$

The wave depth is then given by $x_{fi} = m_{fi}/\rho_s$ and the wave front velocity by $v_{fi} = dx_{fi}/dt \cong x_{fi}/t = m_{fi}/\rho_s t$. It is remarkable that in both LTE regimes ($i = 2$ and 4) the time exponent in m_{fi} is independent of the parameter λ appearing in the energy equation (see eqs. (29,31)).

As usual, the hydrodynamic variables may be written as a product of a dimensional and a dimensionless part:

$$\begin{aligned}
 u_i(m,t) &= u_i(t)U_i(\xi_i), \\
 v_i(m,t) &= v_i(t)V_i(\xi_i), \\
 p_i(m,t) &= p_i(t)P_i(\xi_i), \\
 w_i(m,t) &= w_i(t)W_i(\xi_i), \\
 S_i(m,t) &= S_i(t)S_i(\xi_i),
 \end{aligned}
 \tag{33}$$

where u_i , v_i , p_i , w_i , S_i are products of powers of the governing parameters (except m) with dimensions of velocity, density, pressure, energy per unity mass and energy flux, respectively. As already mentioned, they are called characteristic quantities.

Self-similar solutions for the different regimes may be calculated by introducing the variables in this form into the original system of partial differential equations (4,5,28-31) which then reduces to a system of ordinary differential equations. Solving these equations one can find the dimensionless functions $\tilde{G}_i$ as functions of ξ_i ; they represent the hydrodynamic flow in dimensionless form. We shall not give, however, these functions here but refer to the literature cited above.

Our main interest in this section is not in the spatial profiles of the hydrodynamic variables but in the boundary values, e.g. the pressure at the wave front (denoted by p_f), the heat flux at the vacuum/material boundary (denoted by S_v) or the coordinate of the wave front. They are given (omitting the index i) by $p_f = p(t) \tilde{P}(\xi_f) = p(t) \tilde{P}_f$, $S_v = S(t) \tilde{S}(\xi=0) = S(t) \tilde{S}_v$ and ξ_f . In principle, to obtain $\tilde{P}_f$, $\tilde{S}_v$ or ξ_f , the self-similar solution must be calculated. However, as a rule the boundary values (when finite) of $\tilde{G}_i$ (and ξ_i) are found to be of order unity. For example, for the LTE-AHW regime we found in part III for the case where the heat flux into the wave is constant in time $\tilde{P}_f = 0.70$, $\tilde{S}_v = 2.39$ and $\xi_f = 0.48$ (for $\gamma = 5/4$ and with the normali-

zation $\tilde{T}_V = 1$). The boundary values thus only differ from the characteristic values by a factor of order unity. This difference may be less than the uncertainty introduced by the finite accuracy of the material data.

In the following we shall therefore work not with the exact boundary values of the self-similar solutions but only with characteristic quantities, i.e. we set the boundary values of the dimensionless functions (if finite) equal to one. In other words, our analysis is based on dimensional analysis using the governing parameters of the problem.

In this approach a (characteristic) physical quantity of interest is formed by dimensional analysis from the governing parameters A_1, t and Q (the latter coming from the assumed boundary condition). For Q we consider explicitly a given heat flux S_0 into the wall, a given pressure p_0 in the heated material and a given quality factor $N_0 = \sigma T^4 / S_0$ (for the physical significance of N_0 we refer to the discussion in Sec. II). The subscript 0 is used for the given characteristic quantities in order to distinguish them from the exact boundary values. For completeness we note that Q may also contain a time dependence in the form of a power law; the equivalence of such a case with the usual (time independent) boundary condition has been discussed in Sect. III.

In this work our main interest is in the time evolution of the temperature (entirely equivalent to the time evolution of W). Thus we form a characteristic quantity W_0 from A_1, t and Q for the three choices of Q . For simplicity we abbreviate W_0 (regime 1) by W_1 , etc.

By means of dimensional analysis we find the specific energy density W (equivalent to the temperature) for a constant energy flux S_0 into the wall:

$$\begin{aligned}
 W_1 &= (S_0 t / A_1)^{1/(s+1)} & \text{NEQ-HW,} \\
 W_2 &= (S_0^2 t / A_2)^{1/(\lambda+2)} & \text{LTE-HW,} \\
 W_3 &= (S_0^r t / A_3)^{2/(3r+2s-1)} & \text{NEQ-AHW,} \\
 W_4 &= (S_0^{\mu+1} t / A_4)^{2/(2\lambda+3\mu+1)} & \text{LTE-AHW.}
 \end{aligned}
 \tag{34}$$

For constant pressure p_o we find

$$\begin{aligned}
 w_1 &= p_o / \rho_s & \text{NQ-HW,} \\
 w_2 &= p_o / \rho_s & \text{LTE-HW,} \\
 w_3 &= (p_o^r t / A_3)^{1/(r+s-1/2)} & \text{NEQ-AHW,} \\
 w_4 &= (p_o^{\mu+1} t / A_4)^{1/(\lambda+\mu)} & \text{LTE-AHW.}
 \end{aligned} \tag{35}$$

For a constant quality factor $N_o = \sigma T^4 / S_o$ we find

$$\begin{aligned}
 w_1 &= [(\sigma / K^{4/\delta}) N_o^{-1} t / A_1]^{\delta/(s\delta+\delta-4)} & \text{NEQ-HW,} \\
 w_2 &= [(\sigma / K^{4/\delta}) N_o^{-1} (t / A_2)^{1/2}]^{2\delta/(\lambda\delta+2\delta-8)} & \text{LTE-HW,} \\
 w_3 &= [(\sigma / K^{4/\delta}) N_o^{-1} (t / A_3)^{1/r}]^{2r\delta/(\delta(3r+2s-1)-8r)} & \text{NEQ-AHW,} \\
 w_4 &= [(\sigma / K^{4/\delta}) N_o^{-1} (t / A_4)^{1/(\mu+1)}]^{2\delta(\mu+1)/(\delta(2\lambda+3\mu+1)-8(\mu+1))} & \text{LTE-AHW.}
 \end{aligned} \tag{36}$$

The formal difficulty that N_o is a dimensionless quantity disappears in the last case if the dimensional quantity $N_o K^{4/\delta} \sigma^{-1}$ is considered as the given characteristic quantity.

We note here that if we specify the temperature T_o or alternatively $w_o = K T_o^\delta$ as boundary condition, the corresponding values of S_i , P_i and N_i ($i = 1, 2, 3, 4$) can be obtained directly by inversion of eqs. (34, 35, 36).

IV.4 Border lines between the different regimes

The four relations (34) (and similarly the relations (35) and (36)) represent in a log-log W - t plane (in the figures we shall use the entirely equivalent T - t plane) four families of straight lines with the energy flux S_o as parameter. In principle, there is a fourfold covering of this plane by these lines, corresponding to our four different regimes of hydrodynamic flow. We make now the assumption that at each point of this plane only a single type of flow exists. This has the consequence that the T - t

plane becomes subdivided into several areas, each one corresponding to one hydrodynamic regime and raises the question of the border lines separating the different regimes.

First we consider as an example the practically important border line 2-4 (i.e. LTE-HW/LTE-AHW), see Fig. 9. In the LTE-HW regime a heat wave propagates in a medium of constant density and its characteristic temperature rises with time according to (34.2). However, as time proceeds, expansion of the heated material sets in and the flow changes into the LTE-AHW regime. In this regime the characteristic temperature, according to (34.4), rises more slowly with time owing to the additional cooling connected with the expansion of material. Approximately, the transition from one regime to the other will occur when the temperatures predicted for the two regimes are equal, i.e. at the intersection point of the corresponding S_0 curves.

This consideration suggests that border lines between two regimes be defined by the condition that the value of a certain characteristic physical quantity in the two adjacent regimes be equal on the border line. With the characteristic quantity in the regime i denoted by $g_i(A_i, W, t)$ this condition for the border line 2 - 4 is

$$g_2(A_2, W, t) = g_4(A_4, W, t).$$

This is a relation of the form

$$A_2^{\alpha_{24}} A_4^{\beta_{24}} W^{\gamma_{24}} t^{\delta_{24}} = 1. \quad (37)$$

If $A_2 \neq cA_4$, where c is a dimensionless factor (i.e. if A_2 and A_4 are dimensionally independent and hence the two regimes physically different), the exponents $\alpha_{24}, \dots, \delta_{24}$ are fully determined (and may be readily calculated by dimensional analysis). This follows from the assumption that A_i, W, t are dimensionally independent.

Relation (37) represents the border line 2-4. It should be remarked that it is obviously independent of the choice of g . This has the important consequence that all characteristic physical quantities are equal on such a border line.

We have altogether four regimes with constants $A_1, \dots, A_4$ (each three of them dimensionally independent) and a border line between any two of them may be defined by analogy with (37). This raises the question about the intersection points between the border lines. It is readily seen that the border lines between any three regimes, say 1,2,3, intersect in a single point. Denoting the coordinates of the intersection point by W^*, t^* , one finds by eliminating either W or t from the expressions for the border lines (which are of the type of relation (37))

$$A_1^{\alpha t} A_2^{\beta t} A_3^{\gamma t} t^{*\delta t} = 1,$$

$$A_1^{\alpha w} A_2^{\beta w} A_3^{\beta w} W^{*\delta w} = 1.$$

These relations define a single intersection point, because A_1, A_2, A_3 are dimensionally independent. With four regimes the border lines in our case considered in Sect. IV.5 (not in general) still intersect in a single point. The condition for this is

$$A_1^{\alpha} A_2^{\beta} A_3^{\gamma} A_4^{\delta} = 1.$$

It can be verified by insertion that this condition is satisfied for the choice of $A_1, \dots, A_4$ made in Sec. IV.5 for the case of a gold wall heated by radiation transport. The division of the W - t (or T - t) plane is then particularly simple; it is divided into four regimes by four border lines which all originate in a single point.

Notwithstanding the observation (noted above) that any characteristic physical quantity takes a unique value on a border line, it may be useful to label these border lines with certain properties which are particularly characteristic of the changes in the hydrodynamic flow along such a line. So we have labelled the border line separating the heat wave from the ablative heat wave regime (running from the lower left through the intersection point to the upper right in Fig. 9) by $\rho_{\text{plasma}} = \rho_s$. This means that on this line the characteristic plasma density ρ_{plasma} ($= \rho_1$) in the regime with ablation has decreased from infinity (corresponding to the assumption of an infinitely dense solid in this regime) to the actual density ρ_s of the solid. An alternate condition satisfied on this line is that the characteristic wave velocity has decreased to sound velocity.

The border line running from the upper left through the intersection point to the lower right has been labelled NEQ/LTE and separates the non-equilibrium regimes from the LTE regimes. On the left-hand side of this line, the matter is heated by energy absorption from photons or particles proceeding from the source and its characteristic depth is given by the corresponding (photon or particle) mean free path (or deposition depth), that means $\ell_T = \ell_s$ in this region where ℓ_T is the heat wave depth (or alternatively the gradient length of the temperature profile). On the right-hand side of this border line energy penetrates deeper into the wall by diffusion mechanisms.

It should be emphasized that the division of the W-t plane into regions of self-similar flow, separated by border lines on which the hydrodynamic variables are equal, is conceptually very useful. It shows that the evolution of the hydrodynamic flow occurs as a temporal sequence of self-similar regimes with a clear physical characterization and with scaling laws following immediately from the characteristic parameters of the regime. One can expect that in each regime the true solution follows the dimensional relations (34-36) (within a factor of order unity), only the "kinks" at the border lines (see Fig.10) being smoothed out upon transition from one state to the other.

Writing out the expressions for the border lines one finds for the border line NEQ / LTE regime

$$\begin{aligned} A_1^2 A_2^{-1} W^{2s-\lambda} t^{-1} &= 1 && \text{HW,} \\ A_3^{\mu+1} A_4^{-r} W^{(s-1/2)(\mu+1)-r(\lambda-1)} t^{r-\mu-1} &= 1 && \text{AHW.} \end{aligned} \quad (38a,b)$$

For the border line HW/AHW (i.e. $\rho_{\text{plasma}} = \rho_s$) one finds

$$\begin{aligned} A_4^{-1} \rho_s^{\mu+1} W^{1-\lambda} t &= 1 && \text{LTE} \\ A_3^{-1} \rho_s^r W^{1/2-s} t &= 1 && \text{NEQ} \end{aligned} \quad (39a,b)$$

The four lines intersect at the common point

$$W^* = [A_3^{-2} A_4^2 \rho_s^{2(r-\mu-1)}]^{1/(2s-2\lambda+1)}$$

$$t^* = [A_3^{-2(\lambda-1)} A_4^{2s-1} \rho_s^{2(r+s)(\lambda-1)-2s(\lambda+\mu)+\mu+1}]^{1/(2s-2\lambda+1)}.$$

We also note here that for temperatures higher than those where the radiation pressure is higher than the material pressure, the hydrodynamic equations (4-6) do not appropriately describe the physical processes under consideration. The relation radiation pressure $(= (4/3) \sigma T^4 / c_0)$ = matter pressure defines limiting curves in the W - t plane, given by (c_0 is the light velocity)

$$T = [3K\rho c_0/4\sigma]^{1/(4-\delta)} \quad \text{HW,}$$

$$T = [A_4 K^{\lambda+\mu} t^{-1} (3c_0/4\sigma)^{\mu+1}]^{1/[4(\mu+1)-\delta(\lambda+\mu)]} \quad \text{AHW.}$$

IV.5 Heat penetration into a planar wall: a numerical example

As an example of the application of our theoretical considerations, we consider the radiative heating of a gold wall. In this case (compare Appendix 2) we chose for the constants the same values as in Sec. III:

$$j = 2; \mu = \mu' = 2; v = j + 3 = 5; \delta = 3/2; \lambda = (v + 1 - \delta)/\delta = 3; \\ \rho_s = 19.3 \text{ g/cm}^3; K = 7.4 \times 10^3 \text{ cm}^2 \text{ s}^{-2} \text{ K}^{-3/2}; \delta = 3/2; a = 5.7 \times 10^{-35} \text{ s}^5 \text{ g}^3 \text{ cm}^{-13}.$$

With the choice of these constants the LTE regimes are fully determined. For the NEQ regimes we have to specify in addition the mean free path ℓ_s of the source radiation which depends on the physical nature of the source. Because we do not intend to specify to particular sources in this section but, on the other hand, want to be specific, we set

$$\ell_s = \ell_R$$

i.e. we assume a fictitious source which emits photons with a range equal to the Rosseland mean free path. With this assumption one finds,

by comparing the expressions (1) and (27) for ℓ_R and ℓ_s , respectively, $\ell_o = A_\ell K^{-j/\delta} = 5.96 \times 10^{-21} g_s^{28/3} \text{cm}^{-23/3}$, $s = j/\delta = 4/3$ and $r = \mu' = 2$. These constants determine the NEQ regimes.

The choice of a specific source (even if fictitious) has the advantage that it enables us to illustrate the procedure by which the transition to the LTE regimes can be treated. The particular choice has been made because $\ell_s = \ell_R = \ell_T$ is fulfilled on the border line between the NEQ and LTE regimes (within a factor of order unity) i.e. we obtain by this choice the familiar limit of applicability of the conduction approximation. On the left-hand side of this line, in addition to the equality $\ell_s = \ell_T$ we have also that ℓ_R is always equal to ℓ_T within a numerical factor of order unity. On the right-hand side of the line, ℓ_T becomes larger than ℓ_R and the wave becomes optically thick. Otherwise it must be seen that our main interest in this section is in the LTE regimes; a discussion of source-related effects is given for the laser case in Sec. VII.

In terms of the temperature $T = (W/K)^{2/3}$ the expressions for the border lines are

$$\begin{aligned}
 T &= 1.32 \times 10^{10} t^{1/3} && \text{border line} && 2 - 4, \\
 T &= 4.55 \times 10^{15} t^{4/5} && \text{border line} && 1 - 3, \\
 T &= 2.75 \times 10^{-18} t^{-2} && \text{border line} && 1 - 2, \\
 T &= 7.82 t^{-4/9} && \text{border line} && 3 - 4.
 \end{aligned}
 \tag{40.a-d}$$

The intersection point is located at

$$\begin{aligned}
 T^* &= [(\ell_o \rho_s a^{-1})^{6/7} K^{-1}]^{2/3} = 1.47 \times 10^6 \text{ }^\circ\text{K}, \\
 t^* &= [\ell_o^4 \rho_s^{-3} a^{-5/3}]^{3/7} = 1.37 \times 10^{-12} \text{ s}.
 \end{aligned}
 \tag{41.a,b}$$

The radiation pressure exceeds the material pressure above the lines

$$\begin{aligned}
 T &= 7.97 \times 10^7 \text{ }^\circ\text{K} && \text{HW} \\
 T &= 2.64 \times 10^6 t^{-2/9} && \text{AHW}
 \end{aligned}$$

The boundaries between the different regimes, the intersection point, and the lines $p_R = p_i$ are shown in Fig. 9 for this case.

The curves of constant S_o , p_o and N_o are:

for constant energy flux S_o :

$$\begin{aligned} T_1 &= 3.68 \times 10^3 [S_o t]^{2/7} && \text{NEQ-HW,} \\ T_2 &= 144 [S_o^2 t]^{2/15} && \text{LTE-HW,} \\ T_3 &= 8.67 [S_o^2 t]^{4/23} && \text{NEQ-AHW,} \\ T_4 &= 8.60 [S_o^3 t]^{4/39} && \text{LTE-AHW;} \end{aligned} \quad (42.a-d)$$

for constant pressure p_o :

$$\begin{aligned} T_1 &= T_2 = 3.66 \times 10^{-4} [P_o]^{2/3} && \text{NEQ-(LTE-)HW,} \\ T_3 &= 151 [P_o^2 t]^{4/17} && \text{NEQ-AHW,} \\ T_4 &= 96.9 [P_o^3 t]^{2/15} && \text{LTE-AHW;} \end{aligned} \quad (43.a-c)$$

for constant number N_o :

$$\begin{aligned} T_1 &= 3.38 \times 10^{-17} [N_o^{-1} t]^{-2} && \text{NEQ-HW,} \\ T_2 &= 4.16 \times 10^{-16} [N_o^{-2} t]^{-2} && \text{LTE-HW,} \\ T_3 &= 23.9 \times [N_o^{-2} t]^{-4/9} && \text{NEQ-AHW,} \\ T_4 &= 41 \times [N_o^{-3} t]^{-4/9} && \text{LTE-AHW.} \end{aligned} \quad (44.a-d)$$

We remember here that by inversion of eqs. (42-44) we can obtain S_i , P_i , and N_i ($i = 1,2,3,4$) as a function of the characteristic temperature T_o .

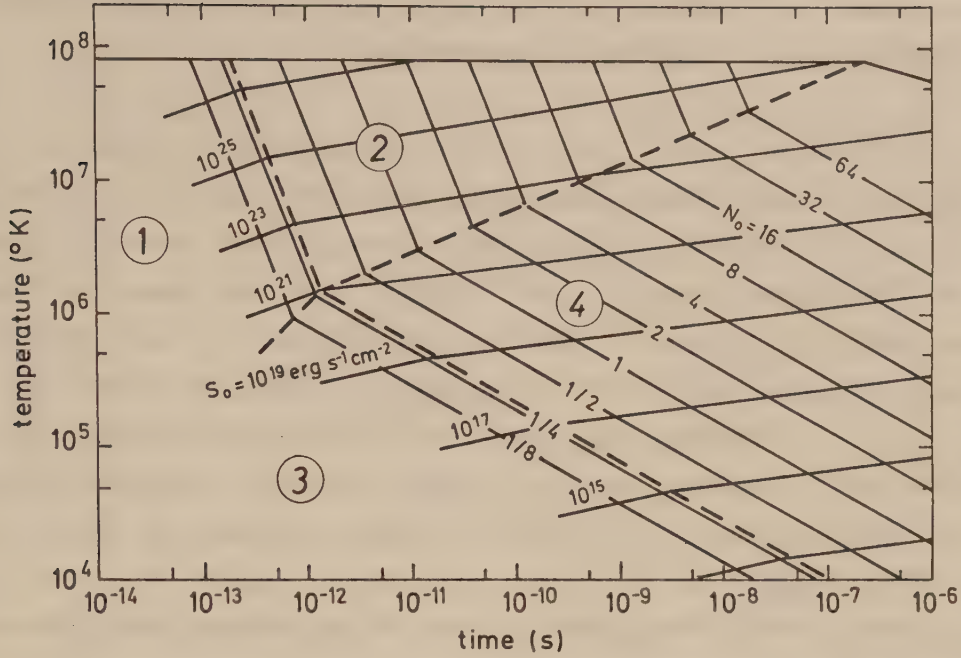


Fig. 10: The characteristic temperature as a function of time for different values of the heat flux S_0 into the wall (from eq.(42)) and lines of constant N_0 (from eq. 44). 1 = NEQ-MW; 2 = LTE-HW; 3 = NEQ-AHW; 4 = LTE-AHW.

Figure 10 shows the T - t plane with the border lines and lines of constant S_0 and N_0 . In the NEQ regimes the lines are only drawn near the $\ell_R = \ell_T$ border line, where the particular case treated here may have physical significance.

Let us first follow a line of constant S_0 . It can be seen that, from left to right, one gets a slower increase of the temperature with time. This is because with the chosen values of the exponents λ , μ , r and s the wave front runs faster in the right than in the left regimes. In the NEQ-HW regime the wave front advances because the mean free path of the source radiation increases with temperature, i.e. as the wall material is heated. In the NEQ-AHW regime expansion of the heated material reduces its density and, because the mean free path increases with decreasing density, leads to even faster propagation of the wave.

In the LTE regimes the onset of radiative conduction further enhances the wave speed, with the LTE-AHW wave again propagating faster than the LTE-HW owing to the effect of expansion.

For $S_0 < 10^{21} \text{ erg s}^{-1} \text{ cm}^{-2}$, i.e. as long as the lines of constant S_0 run below the intersection point, they pass from the ablative NEQ regime to the ablative LTE regime. In this case heating of the material is accompanied by expansion. For $S_0 > 10^{21} \text{ erg s}^{-1} \text{ cm}^{-2}$ the lines pass initially from the NEQ-HW regime to the LTE-HW regime; in both regimes the waves move through material of solid density. When the border line 2 - 4 ($\rho_{\text{plasma}} = \rho_s$) is reached, the character of the wave changes: a shock wave forms and expansion of the heated material becomes important. After the border line is crossed the flow continues in the LTE-AHW regime; this regime is thus the asymptotic regime for any source intensity except when the regime is entered where radiation pressure is larger than matter pressure.

As was discussed in Sec.II, it is important that the flow reaches the LTE regime rapidly. It is now seen that this regime is established the more rapidly the higher the source intensity (and temperature) is. At least for the idealized conditions considered here, the transition occurs even for a relatively low intensity of $10^{19} \text{ erg s}^{-1} \text{ cm}^{-2}$ and a corresponding temperature of $\sim 5 \times 10^5 \text{ K}$ within a few tens of picoseconds, a time much shorter than typical pulse durations of laboratory power sources.

The lines of constant N_0 run parallel to the $\ell_R = \ell_T$ border lines (3-4 and 1-2) because they are defined by the same parameter combination (apart from a dimensionless number). It is seen that N_0 becomes larger than one as the wave develops into the LTE regime.

Figure 11 shows lines of constant pressure p_0 . In the HW regimes the lines of constant pressure correspond to lines of constant temperature because the density is constant. In the AHW regimes the temperature must increase with time in order to maintain a constant pressure; this is of course a consequence of the decrease in density due to expansion. If the temperature is maintained, the pressure drops with time. The variation of the pressure along lines of constant S_0 or N_0 can be found by superposing Figs. 10 and 11.

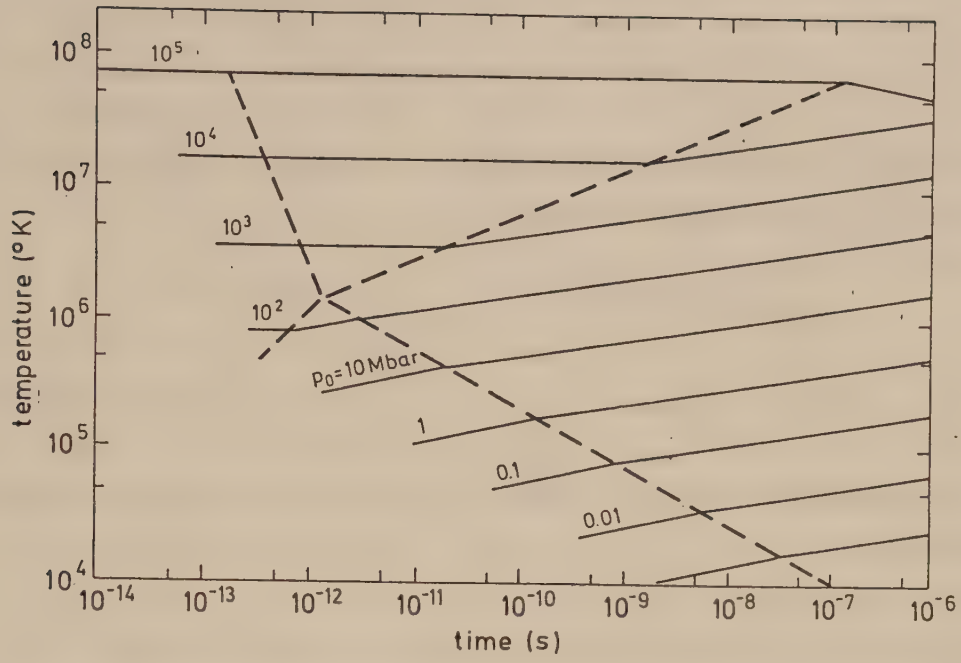


Fig. 11: Isobars in the T-t plane as given by eq. (43).

V. The generation of a radiation field in a cavity

So far we have only considered the planar problem of the interaction of the radiation field with the wall. The planar treatment is appropriate up to the time when the ablated material has expanded a distance small compared with the radius of the cavity. With the characteristic plasma velocity given by $W^{1/2}$ this condition is $t = RW^{-1/2}$. With $W = KT^{3/2}$ it becomes for gold (in cgs units and $^{\circ}K$)

$$T = K^{-2/3} R^{4/3} t^{-4/3} = 2.6 \times 10^{-3} R^{4/3} t^{-4/3}. \quad (45)$$

Again this relation defines a line in the T - t plane. For example, for $T = 10^6$ $^{\circ}K$, $R = 10^{-1}$ cm one finds $t = 3.7 \times 10^{-8}$ s. For a power source with a characteristic time of $\sim 10^{-9}$ s a planar wall treatment is therefore well justified.

Although the wall problem may be treated in planar geometry, the finite size of the cavity must be considered with respect to its energy balance. This is the purpose of this section. For our discussion we have in mind a cavity with characteristic radius R , surrounded by solid gold walls. The role of the particular shape of the cavity and its influence on the isotropy of the radiation field is not discussed. Nevertheless it may be noted that a spherical cavity is particularly simple in this respect because it has the property that the radiation from an optically thick surface element uniformly illuminates the inner surface.

In the energy balance of the cavity we take into account the losses through a hole in the cavity wall and the energy stored in the radiation field. We denote by F the power released into the cavity. The energy balance of the cavity is then

$$F = a_w S_{HW} + a_h \sigma T^4 + dQ_R/dt, \quad (46)$$

where $a_w S_{HW}$ is the energy diffusing into the wall per unit time, $a_h \sigma T^4$ is the external loss through the hole and Q_R is the energy content of the radiation field inside the cavity; a_w is the inner surface area of the cavity and a_h the area of the hole. Q_R is given by

$$Q_R = (4/3) \pi R^3 \sigma_o^{-1} T^4$$

for a black-body radiation field.

We now discuss the relative importance of the three terms in eq. (46). Let us assume first that

$$F \cong a_w S_{HW}, \quad (47)$$

i.e. the energy loss through the hole and the storage of energy in the radiation field are negligible. With a_w and F specified, the heat flux $S_{HW} \cong S_o$ into the wall is fully determined. In this case the cavity behaves as shown in Fig. 10 for a given S_o . It corresponds to a closed cavity. All the energy supplied by the source is forced to flow immediately into the wall.

Let us now assume that the cavity has a hole with an area a_h . In this case the temperature cannot rise indefinitely, but will stabilize when the losses through the hole become comparable with the power supplied by the source. The limiting temperature (reached only asymptotically) is given by

$$T_{lim} = (F/a_h \sigma)^{1/4}. \quad (48)$$

Once the temperature stabilizes near T_{lim} , the heat flux into the wall will decrease for the LTE-AHW regime in proportion to $t^{-1/3}$ and for the LTE-HW regime in proportion to $t^{-1/2}$ (this time dependence can be directly obtained by inversion of eqs. 42 b and d, solving S as a function of T_o and t)

Let us now consider the case when the radiation field in the cavity is important as an energy reservoir. If the third term in eq. (46) is dominant, one has

$$F \cong dQ_R/dt = (16/3)\pi R^3 \sigma c_o^{-1} T^3 dT/dt. \quad (49)$$

Integration gives

$$T = (3Fc_o t/4\pi R^3 \sigma)^{1/4} = (3S_o c_o t/R\sigma)^{1/4}. \quad (50)$$

If during the heating the temperature comes near to or even exceeds this value, the storage of energy in the radiation field must be taken into account, otherwise it may be neglected.

In our experiments the source delivers an energy of about 100 J in 300 ps into cavities with a surface of about 10^{-2} cm^2 . This gives a source energy flux into the wall of about $10^{20} \text{ erg/cm}^2 \text{ s}$. We have used cavities with a radius of $150 \times 10^{-4} \text{ cm}$ and $500 \times 10^{-4} \text{ cm}$ and with a hole of about $25 \times 10^{-4} \text{ cm}$ and $100 \times 10^{-4} \text{ cm}$ radius, respectively. With the available energy flux and the type of cavities used, we can see that the dominant term in eq. (46) is the loss $a_w S_{HW}$ into the wall. In fact, assuming first that the losses into the wall are approximately equal to the total laser power, we can expect a temperature of 10^6 K from by eq. (42) and Fig. 10. For this temperature we can estimate the energy flowing into the black-body radiation field as $dQ_R/dt \cong Q_R/t \cong 10^{16} \text{ erg/s}$ and the energy flowing through the hole as: $a_h \sigma T^4 \cong 2 \times 10^{16} \text{ erg/s}$. Comparing them with the source energy flux of order 10^{20} erg/s , we see that the power required for the build-up of the radiation field and the power lost through the hole are indeed negligible fractions of the absorbed laser power and the temperature in the cavity is determined mainly by the power loss into the wall.

It was said at the beginning of this work that one of the most important characteristics of the cavity is the property to produce a very isotropic radiation field inside it. It will be useful to introduce a relative measure for the anisotropy of the thermal radiation flux inside the cavity. It is clear that the isotropy of the radiation field depends, at least at the beginning of the heating process, on the uniformity of energy deposition by the energy source. Obviously, if all the wall elements of the cavity were uniformly irradiated by the source, the temperature of the wall should increase also uniformly and the radiation would be isotropic. On the other hand, the existence of a hole in the wall (equivalent to a wall element with zero temperature) will be a cause of permanent anisotropy for the radiation field. It is expected, however, that for $N \gg 1$ the nonuniformity in the wall heating will be smeared out and the isotropy of the radiation field will be enlarged even in the presence of holes.

Because the initial uniformity in the distribution of the external energy source along the cavity is considered as unknown and depends strongly on the physical mechanisms of absorption and thermalization of the source energy, we will analyze a limiting case of strongest possible asymmetry. We consider two plane surfaces as shown in Fig. 12, where only one surface receives source radiation with intensity S_s , whereas the

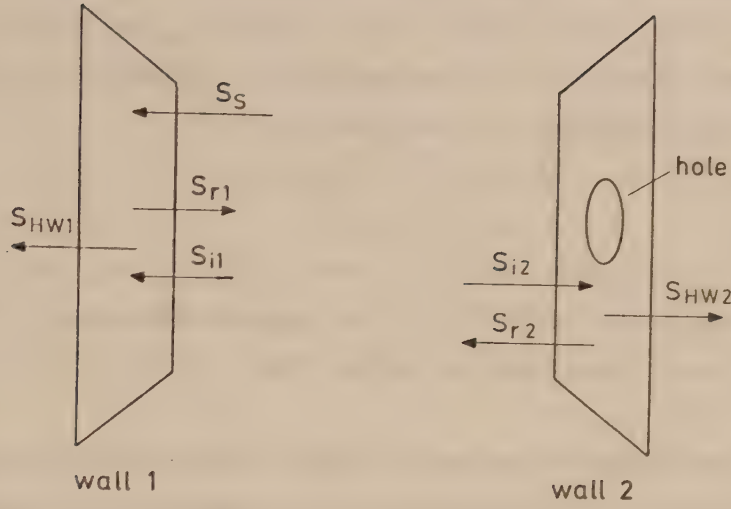


Fig. 12: Schematic representation of the radiative energy exchange between two planar walls. The wall 1 is directly irradiated by an external source with a flux S_s , emits a radiant emittance S_{r1} and receives a radiant emittance S_{i1} . An energy flux equal to S_{HW1} is absorbed by the wall. The wall 2 emits a radiant emittance flux S_{r2} , receives a radiant emittance S_{i2} and absorbs a flux S_{HW2} . The wall 2 has a hole with area a_h . Both walls have an area equal to a_w .

other is not irradiated, and has in addition a hole with area a_h . We suppose that each wall receives a thermal radiant emittance S_i , reemits a thermal radiant emittance S_r and absorbs an energy flux S_{HW} due to losses into the wall. The second wall can be heated only by the radiation flux reemitted by the first wall. We assume that both walls have an area equal to a_w . From energy conservation it holds that:

$$S_{r1} = S_{i2} \quad (51)$$

$$S_{i1} = S_{r2} (1 - a_h/a_w) \quad (52)$$

$$S_{i2} = S_{HW2} + S_{r2} \quad (53)$$

We define in analogy with Sec. II the quality factor N as $N = S_r/S_{HW}$.

A good relative measure of the asymmetry or anisotropy of the radiation field between both surfaces will be given by:

$$\Delta = (S_{r1} - S_{i1})/S_{r1}$$

We note that $S_{r1} > S_{i1}$ and $1 > \Delta > 0$ because the wall 1, being heated by the source has a temperature higher than the wall 2.

For a real, approximately spherical cavity the anisotropy in the internal radiation field will be smaller than Δ , whereas for a single wall the anisotropy is maximal and equal to 1.

Making use of (51), (52) and (53) we find that

$$\Delta = 1 - (1 - a_h/a_w) / (1 + S_{HW2}/S_{r2})$$

or, with the definition of N :

$$\Delta = 1/(N+1) + N(a_h/a_w)/(N+1)$$

We see that the anisotropy has two sources: the first one (due to the first term) is given by the difference in temperatures between both walls and tends to zero when $N \rightarrow \infty$. The second term appears due to the presence of the hole and remains almost constant independent of the value of N .

N can be calculated as a function of the external energy flux (or cavity temperature) and time using the results in Sec. III and IV as long as the anisotropy is slight.

Another important question is about the spectral distribution of the radiation field inside the cavity. It is clear that a cavity in thermal equilibrium must radiate as a perfect black-body. Our cavities, on the other hand, do not reach complete thermal equilibrium due to the continuing energy losses into the wall and the spectral distribution of the radiation field is then determined by the optical thickness of the heated material.

It can be shown using the stationary radiation transfer equation /2/ that the spectral intensity I_v radiated by a wall element with thickness d is given by

$$I_v = I_{vp} [1 - \exp(-\int_{x=0}^d \kappa_v dx)]$$

where I_{vp} is the black-body intensity distribution and κ_v is the absorption coefficient at the frequency v ; κ_v is a function of the local density and temperature.

For a step-like temperature profile like those appearing in non-linear diffusion processes, we approximate the integral by

$$\int_{x=0}^d \kappa_v dx = \kappa_v(T, \rho) \cdot d$$

where now $\kappa_v(T, \rho)$ is a mean value characterized by the material temperature T and mean density ρ and d is the thickness of the heated material. For $d \gg 1/\kappa_v$, the spectral distribution approaches the black-body distribution.

Our comparison of experiment and theory will be based on the spectrally integrated emittance I , that is

$$I = \int_{v=0}^{\infty} I_v dv$$

Integrating the above expression for I_v over frequency, expanding the exponential into a Taylor series in $(\ell_p^{-1} - \kappa_v)$ (where ℓ_p is the Planck mean free path for radiation) and neglecting terms of higher than second order, one obtains

$$I = (1/\pi)\sigma T^4 [1 - \exp(-\int_{x=0}^d dx/\ell_p)]$$

Note that ℓ_p is a function of the temperature T and density ρ of the heated material.

Again the integral is approximated by

$$\int_{x=0}^d dx/\ell_p \cong d/\ell_p \cong d/\ell_R \quad ,$$

in addition it has been assumed that ℓ_p is approximately equal to ℓ_R , the Rosseland mean free path. It can be shown that for the LTE cases analysed in Sec. IV the characteristic ratios d/ℓ_R and $\sigma T^4/S_{HW} = N$ are equal because they are defined by the same combination of dimensional quantities. We can write therefore $d/\ell_R = N$. With this substitution we find

$$I \cong (1/\pi)\sigma T^4 [1 - \exp(-N)] \quad (53.a)$$

and we see that the approach to the black-body spectrum is also governed by the parameter N , the quality factor of the cavity.

VI. Experimental set-up

VI.1 Scope of the experiments

During the course of this thesis we performed experiments where small gold cavities were heated by a pulsed laser beam. No such experiments have been described so far in the literature. This enables us to compare at least some of our theoretical predictions with experimental results.

As we have emphasized already in Sec. II a simple and accurate description of the heating can be obtained only in the limit $N \gg 1$ i.e. if a high quality factor N of the cavity is obtained. In this limit a true equilibrium between radiation and matter is established, the radiation spectrum becomes a Planck spectrum (independent of the properties of the wall), and the frequency averaging procedure used in the approximation of radiation heat conduction leads to accurate results. On the basis of our theoretical results we have calculated that values of $N \cong 3-10$ require pulse energies in the range 10^3-10^6 Joule.

Unfortunately the pulse laser available for the experiments delivers a pulse energy of only 10^2 J. In this case we can expect not more than about $N \cong 1$. Thus we are faced in our experiments with a rather complicated situation: a non-Planckian spectrum, carrying the signature of the wall material, and a marginal accuracy of the theoretical predictions. On the other hand it is encouraging that the expected conditions are at least at the beginning of the range of interest.

As a first step towards the experimental verification of cavity heating we concentrate on the absolute measurement of the radiation field in the cavity by observing it through a small hole in the cavity wall. Though spectrally resolved measurements are made, the major emphasis is on the spectrally integrated flux (which is the quantity to be compared to theory) rather than on details of the spectrum. Fortunately the emitted spectrum is found to be close enough to a Planck spectrum that such a comparison is meaningful.

In this section we describe the instruments used in the experimental set-up and the procedure by which the radiation from the interior of the cavities was registered and analyzed.

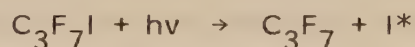
VI.2 The iodine laser

The energy source used in our experiments was the high power iodine laser ASTERIX III at the Max-Planck-Institut für Quantenoptik (MPQ), Garching /17/.

It consists mainly of a pulsed, mode-locked oscillator and four amplifiers, generating a light pulse of about 300 ps duration with an energy up to 300 Joules at the output of the last amplifier. The wavelength of the radiation is 1.3 μm .

The laser medium is a mixture of gaseous iodides and argon with a composition ranging between 100 torr $\text{C}_3\text{F}_7\text{I}$ /400 torr argon in the oscillator and 6 torr $\text{C}_3\text{F}_7\text{I}$ /3 atm argon in the last amplifier. The argon gas is added for pressure broadening.

The population inversion of the laser medium is obtained by flash pumping, producing the reaction



where I^* means an excited state of the iodine atom /17/.

The laser transition is the magnetic dipole transition between the $^2\text{P}_{1/2}$ and $^2\text{P}_{3/2}$ states of the iodine atom, that is $^2\text{P}_{1/2} \rightarrow ^2\text{P}_{3/2}$.

The use of a gaseous laser medium has some advantages over Neodymium glass lasers with similar power characteristics. For example, the relatively high repetition rate of one shot each 15 minutes is possible only in a gas laser with recycling and external cooling of the laser medium. Also, in contrast to glass lasers, the laser is operated in the saturated regime with an advantage in efficiency. Absence of self-focusing effects due to the low nonlinear refraction index of the gaseous medium allows a good focusibility of the beam.

The laser beam can be frequency doubled by a KD*P crystal with a conversion efficiency of about 50 % and frequency tripled with 30 % conversion efficiency /18/. The beam is focused by means of an aspheric lens

of 40 cm focal length. The target is located inside the target chamber, where the pressure is maintained below 10^{-4} millibar. The focus diameter is about $60\text{ }\mu\text{m}$ /19/.

In the target chamber we have also a spectrometer used to analyze the spectrum of the radiation emitted by the cavity and an x-ray pinhole camera looking at the target.

The pinhole camera, carrying a pinhole/filter combination (typically a $20\text{ }\mu\text{m}$ diameter pinhole in combination with a $3\text{ }\mu\text{m}$ thick aluminium filter) produce pictures of the target with a one to one magnification factor on film. It was used to verify the spatial origin of the radiation seen by the spectrometer and the correctness of the laser focusing into the cavity.

Figure 13 shows schematically the experimental set-up.

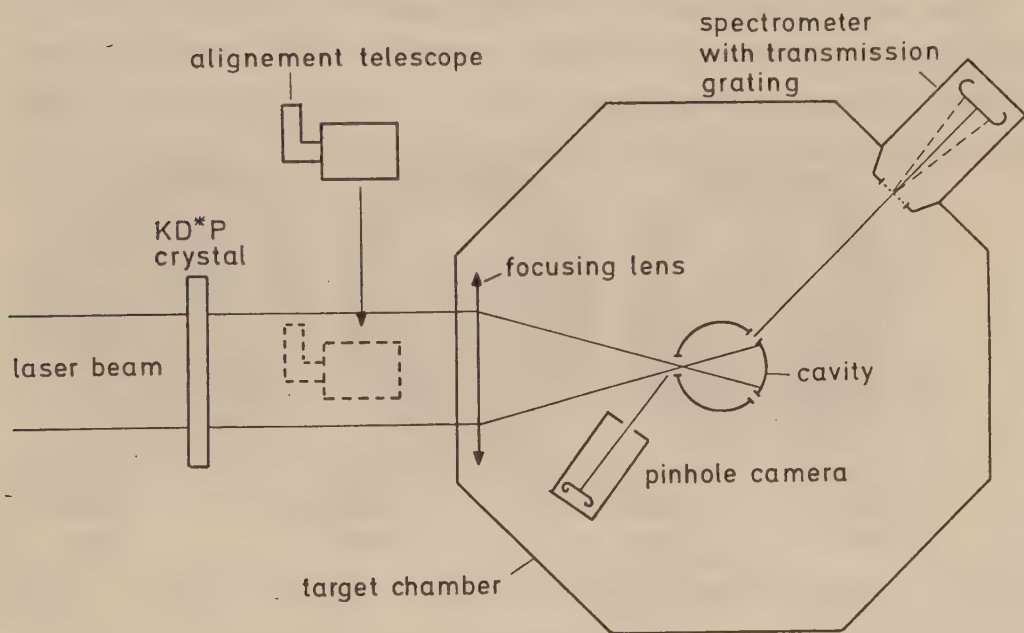


Fig. 13: The target chamber. Shown are the laser beam, the KD*P crystal, the alignment telescope, the focusing lens, a cavity target and two diagnostics: a pinhole camera and the spectrometer with transmission grating.

VI.3 The cavities

Two types of small hollow gold spheres were used as targets in the experiment. A target of the first type is shown in Fig. 14.a. It is a 1 mm diameter sphere with three apertures, made of two hemispheres glued together at the equator. On the front side the entrance hole for the laser beam can be seen. On the rear side in the same horizontal plane are two smaller holes conceived for "looking" at the radiation inside the cavity. The wall thickness is about 40 μm . The second type of cavity is smaller, only 300 μm in diameter and has a wall thickness of 2 μm . Fig. 14.b shows a photography of such a sphere, made with a scanning electron microscope. They are commercially fabricated by gold deposition on glass microballons and posterior dissolution of the residual glass by Firma Dr. Johannes Heidenhain, Traunreut,

It should be noted that through the diagnostic holes wall elements are seen by the spectrometer which are not directly irradiated by the laser beam. The nominal parameters of each type of cavity are given in Table VI. Because the dimensions varied somewhat from piece to piece, each cavity was measured individually and its exact dimensions taken into account for the analysis of the experimental data.

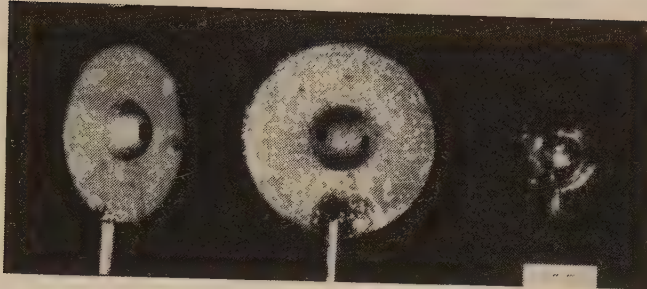
Table VI.

Nominal parameters of each type of cavity

	cavity diameter (μm)	entrance hole diameter (μm)	diagn. hole diameter (μm)	wall thickness (μm)
big cavity	1000	300	200	~ 40
small cavity	300	150	50	2

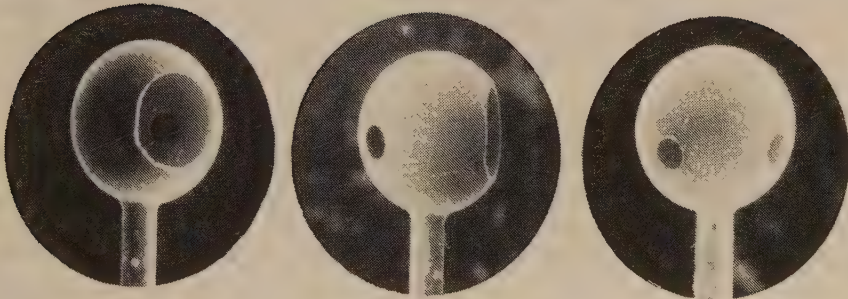
Both types of cavities were held inside the target chamber by a fine wire at the focus of the laser beam. In the realization of the experiment, it was necessary to focus the laser beam into the center of the entrance

a) big cavity



1mm

b) small cavity



300 μ m

Fig. 14: a) A big cavity about 1 mm in diameter. The left photograph shows the front side with the entrance hole, the middle the rear side with two diagnostic holes, and the right a cut through the cavity. b) A small cavity, about 300 μ m is diameter. The left photograph shows the front side with the entrance hole, the middle a lateral view, and the right the rear side with two diagnostic holes.

hole of the cavity. The following procedure was adopted: the axial location of the focus was found by a routine method using a Hartmann plate whereas the lateral position was determined with the help of a telescope placed in front of the focusing lens as shown in Fig. 13. First, a polaroid film was used as a target and exposed by a very attenuated beam, thus generating a burn mark at the focal position. For the second step, the graticule of the eyepiece was centered onto this burn mark. Finally the polaroid film was removed, the target brought at its place and its entrance hole centered on the graticule of the telescope. This procedure is shown schematically in Fig. 15.

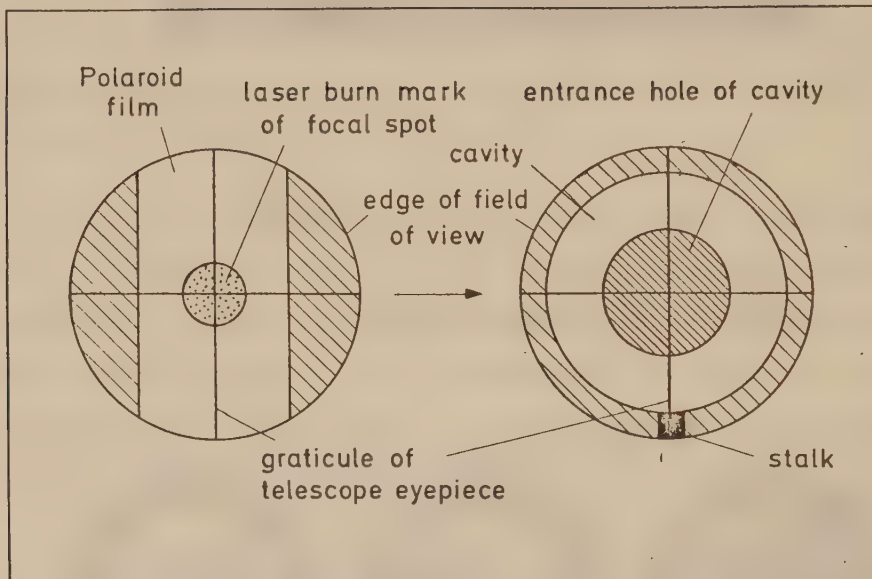


Fig. 15: Schematic representation of the procedure used to center the entrance hole of the cavity onto the beam focus.

An important point to be answered before the beginning of the experiment concerned the possibility that energy deposition inside the cavity might be prevented by a fast closing of the entrance aperture for the laser light before the end of the laser pulse. In fact because the entrance hole has about the same diameter as the focal spot, it is possible that laser heating of the wall around the hole leads to the formation of an expanding plasma. Due to the low value of the critical density for the laser wavelength (about 1/100 of the solid state density) a reflecting layer of plasma could form in the entrance hole. Obviously in this

case part of the laser energy would be reflected and the cavity would not reach the maximum possible temperature.

Fig. 16 shows the experimental arrangement used to clarify this problem by measuring, as a function of the hole diameter, the transmission through holes in thin gold strips. 1.5 meter behind the target a light diffusing screen was placed, which scatters the light transmitted without deflection. Part of the scattered light was measured with a detector calibrated in situ with no target present. For the detection of light deflected by the pinhole, a calibrated Ulbricht sphere was located immediately behind the hole. It covers the solid angle outside the cone of the undeflected light beam.

VI.4 The transmission grating spectrometer

The spectrum of the radiation emitted through the diagnostic hole was measured by means of a soft x-ray spectrometer. It consists of a gold transmission grating and a camera for x-ray film. The spectrometer is shown schematically in Fig. 17. The spectral measurement is based on the grating dispersion formula, which for small diffraction angles can be written as

$$m\lambda = a \sin \theta \cong ay/(R-r)$$

where m is the spectral order, λ the wavelength, a the grating constant, θ the diffraction angle, R the distance cavity-film, r the distance cavity-grating and y the coordinate in the dispersion direction as shown in Fig. 17.

We have used transmission gratings originally developed at the Max-Planck-Institut für Extraterrestrische Physik, /20/ and fabricated commercially by Firma Dr. Johannes Heidenhain, Traunreut. Because for the wavelength range of interest, between 10 Å and 100 Å, no optical imaging system exist, we have used the grating in combination with a pinhole camera. Pinhole and grating are shown as a single unit in Fig. 18a and b. The use of such a pinhole-grating combination has the advantage, that in addition to spectral resolution, spatial resolution is

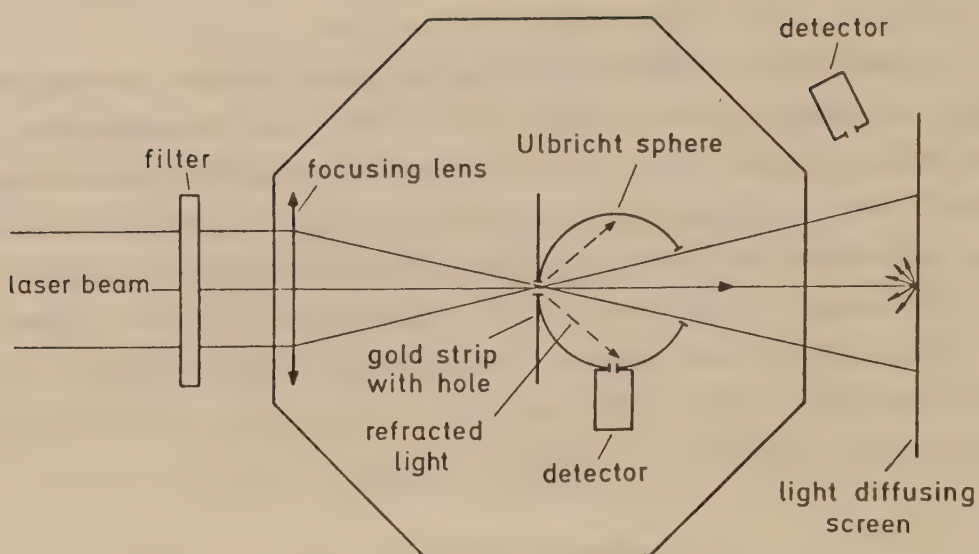


Fig. 16: Experimental arrangement to measure the energy transmission through holes of different diameters.

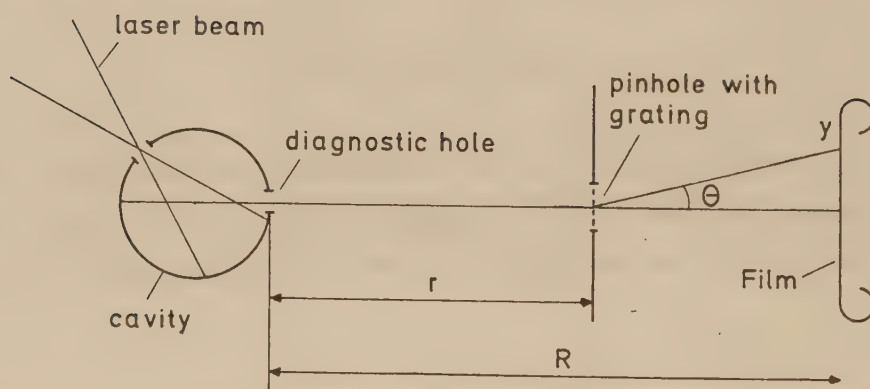


Fig. 17: Geometry and position of the spectrometer with respect to the diagnostic hole and laser beam. The spectrum was recorded on x-ray film. The distance diagnostic hole-pinhole was $r = 30$ cm, the distance diagnostic hole-film $R = 60$ cm.

also obtained. The light transmitted in zero order forms a pinhole picture of the radiation source. For the diffracted light spatial resolution is maintained in the direction perpendicular to the spectrum. We have considered and used initially four types of pinhole-gratings whose characteristics are given in Table VII.

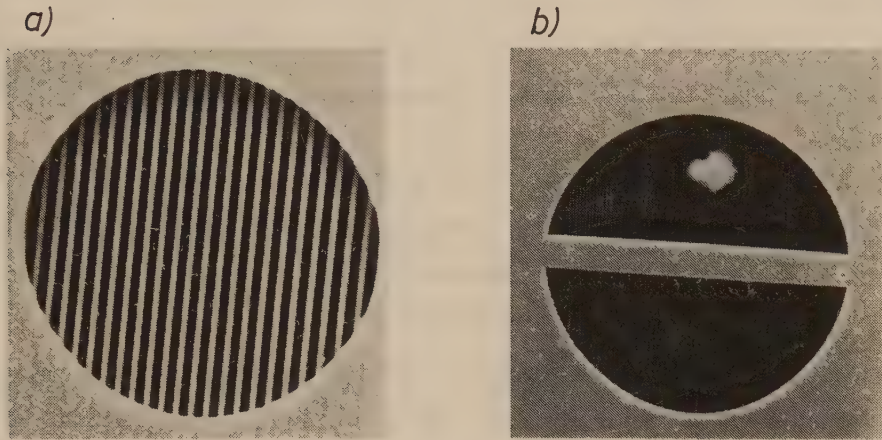


Fig. 18: Pinhole-grating combinations

a: a good sample of 25 μm in diameter and 1000 lines/mm (type A).

b: an unsatisfactory sample of 50 μm in diameter and 2000 lines/mm (type D). This grating shows non-uniformities in the grating bars. The supporting bar in the middle of the pinhole and a dust grain can be seen.

Analysis of test spectra showed that the spectra produced by pinholes of type A and B (1000 lines/mm) were in mutual agreement and much more brilliant than those produced by pinholes of type C or D (2000 lines/mm). The latter did agree neither with each other nor those produced by type A and B. In addition these gratings showed a poorer wavelength resolution. Inspection of the different types of pinhole-gratings with light and electron scanning microscope showed for some pieces, in particular most of the 2000 lines/mm gratings, irregularities in

Table VII
Different types of pinhole-grating combinations

type	lines/mm	pinhole diameter (μm)
A	1000	25
B	1000	50
C	2000	25
D	2000	50

the grating structure, the lines being not equidistant or bent. This failure in the grating uniformity produces a background of scattered radiation and a loss of spectral resolution.

It was found necessary to preselect the gratings and to discard all gratings with 2000 lines/mm. In Fig. 18.a we show a satisfactory sample of type A and in Fig. 18.b an unsatisfactory one of type D. Note that the 50 μm diameter pinholes of type B and D have a supporting bar perpendicular to the grating lines as seen in Fig. 18.b. Predehl /20/ has calculated and measured the absolute energy efficiency of the different spectral orders taking into account the finite transmission through the grating bars. This efficiency is defined as the fraction of the incident radiation energy with wavelength λ that goes into the respective spectral order. We show in Fig. 19 Predehl's result for the 1000 lines/mm grating. For thick bars the efficiency is determined by the bar-to-slit width ratio. In our case this ratio is one, with the consequence that even orders are suppressed. This can be seen in Fig. 19 where the efficiency into the first order is about 10 %, whereas for the second order it reaches only 0.3 % in the wavelength interval of interest between 10 Å and 100 Å. The appearance of a weak second order is due to the finite bar thickness.

The efficiency into the third order is less than 1 % and still lower for higher orders. Furthermore, in the interval of interest these efficiencies are practically independent of the wavelength. For these reasons we neglect higher order contributions in the following and take a constant first order efficiency equal to 10 %, independent of wavelength.

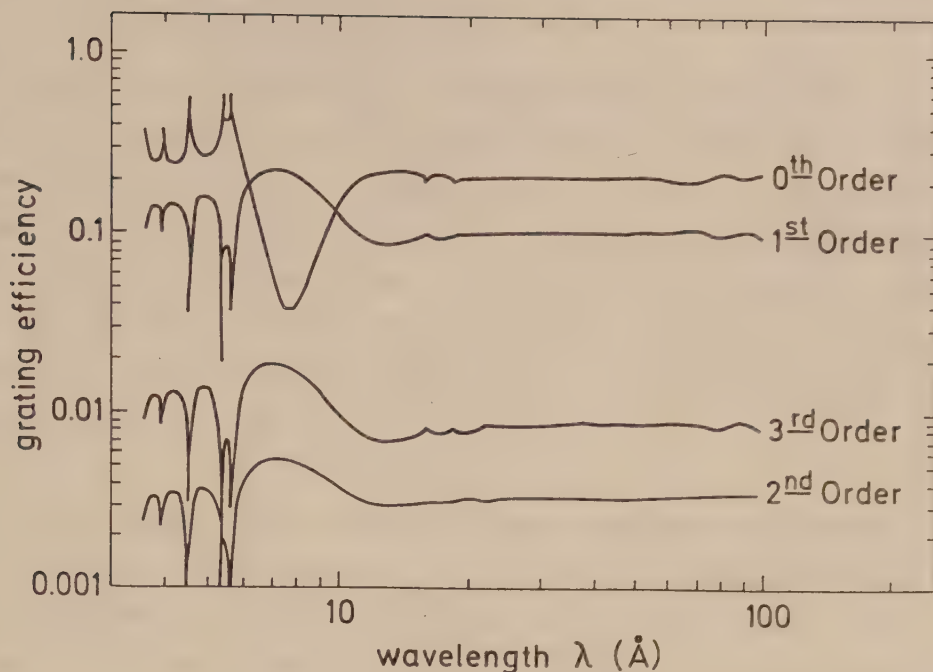


Fig. 19: Grating efficiency into the different orders after Predehl et al. /20/. It is seen that in the wavelength interval 10-100 Å the efficiency into the first order is constant and about 10 %, into the second and third order about 0.3 % and 0.9 %, respectively.

For a point radiation source located at infinity the wavelength resolution $\Delta\lambda$ for a grating (due only to diffraction effects) is given by

$$\Delta\lambda = \lambda/mN = a\lambda/mp \quad (54)$$

where N is the number of grating lines effectively used in the experimental arrangement, and p is the aperture of the opening placed in front of the grating. The real spectral resolution must become poorer due to the finite size of the pinhole and radiation source (i.e. diagnostic hole) and the finite pinhole-to-source distance. In fact, we must add to the spectral resolution given by (54), the finite size of the projections of the pinhole and diagnostic hole onto the film plane. In Appendix 3 we calculated this geometrical effect on the wavelength resolution. It is

$$\Delta\lambda \cong apR/(R-r)r + ad/r$$

where d and p are the diagnostic hole and pinhole diameters, respectively. The geometrical effect dominates the wavelength resolution up to 200 Å wavelength, except for the combination of small cavities (diagnostic holes) with small pinholes.

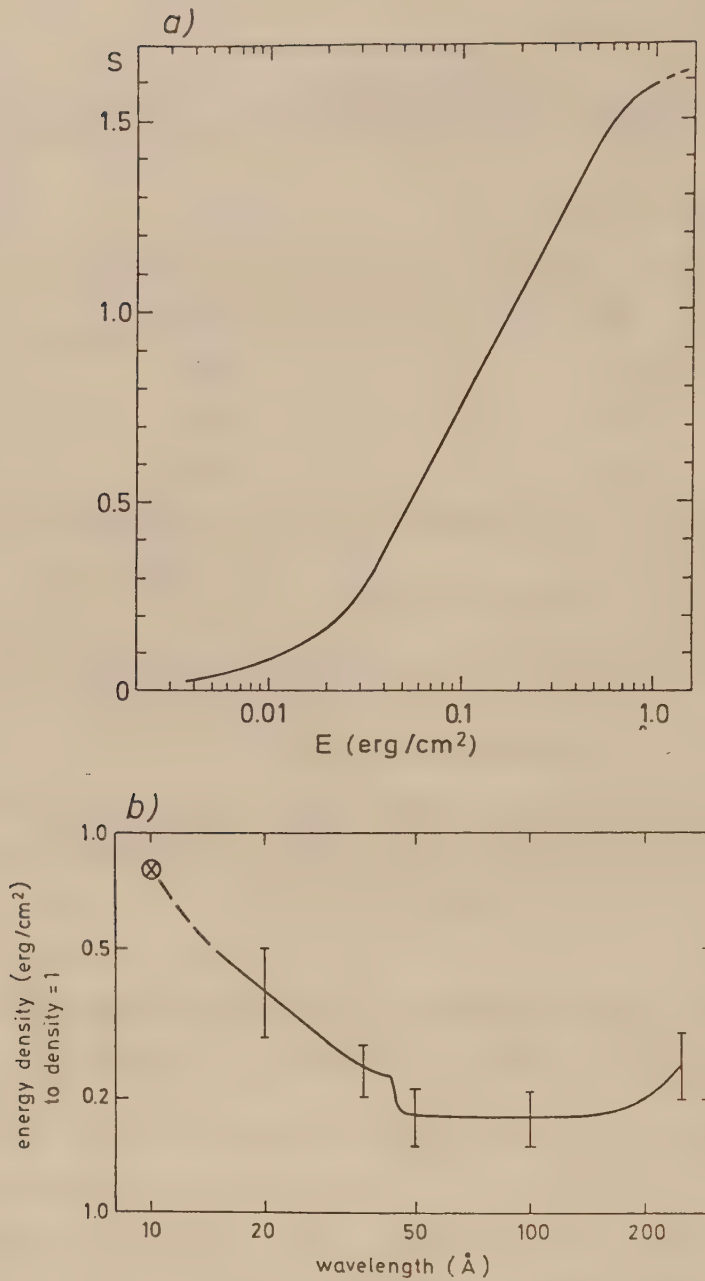


Fig. 20a: Film density S as a function of the energy density for a wavelength of 80 Å. After T. Kishimoto /21/ for Kodak spectroscopic film 101-01.

b) Energy density E_λ needed to attain a film density of one as a function of the radiation wavelength. After T. Kishimoto /21/ for Kodak spectroscopic film 101-01.

We conclude this section by noting that we used Kodak Spectroscopic Film (0650) type 101-01. The density and spectral sensitivity curve of this film are shown in Fig. 20a and b. They were measured by T. Kishimoto /21/ using a calibrated laser-plasma radiation source.

VI.5 Experimental results

As pointed out previously, a difficulty arises due to the possibility of a premature closing of the entrance hole for laser light caused by the formation of a reflecting plasma layer. In order to avoid this effect (by an adequate choice of the entrance hole diameter), we performed, before beginning with the fabrication of the cavities, a series of measurements with the aim to obtain data for the energy transmission through different holes as a function of the hole diameter. The experimental arrangement was shown in Fig. 16.

Fig. 21.a shows the undeflected and the total transmitted light as a function of the hole diameter for $\lambda = 1.3 \mu\text{m}$ called 1 ω light. It is seen that for holes larger than 200 μm in diameter the total transmission exceeds 90 %, for holes of 150 μm in diameter the transmission has gone down to about 50 % with important deflection effects. For holes smaller than 100 μm in diameter the transmission of laser light is already below 10 %. Fig. 21.b shows the transmission through pinholes for $\lambda \cong 0.4 \mu\text{m}$ (3 ω light). We see now that the transmission of 3 ω light increases more rapidly with the hole diameter, reaching about 85 % at 150 μm diameter.

This may be due to a suppression of laser (stray) light in the periphery of the focal spot in the conversion step. In addition the energy carried by the 3 ω pulse is only 30 % of the energy available in a typical 1 ω shot; thus the material around the hole receives a lower intensity and does not expand as rapidly. Furthermore the critical density is 9-times higher in this case, allowing the laser light to penetrate through a denser plasma layer.

These measurements suggest that for the big cavities with an entrance hole of 300 μm diameter we can expect nearly 100 % transmission for both wavelengths whereas only 50 % of the laser energy would be transmitted through the 150 μm diameter entrance hole of the small cavities for 1 ω and 85 % for 3 ω . It must be kept in mind, however, that the measurements with gold strips give only an upper limit for the transmission through

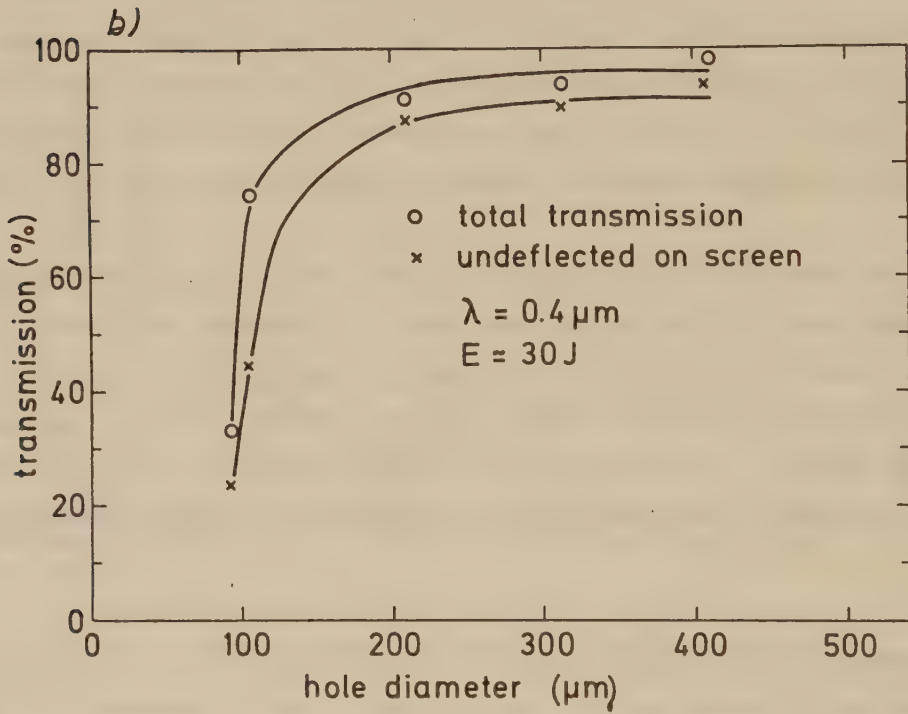
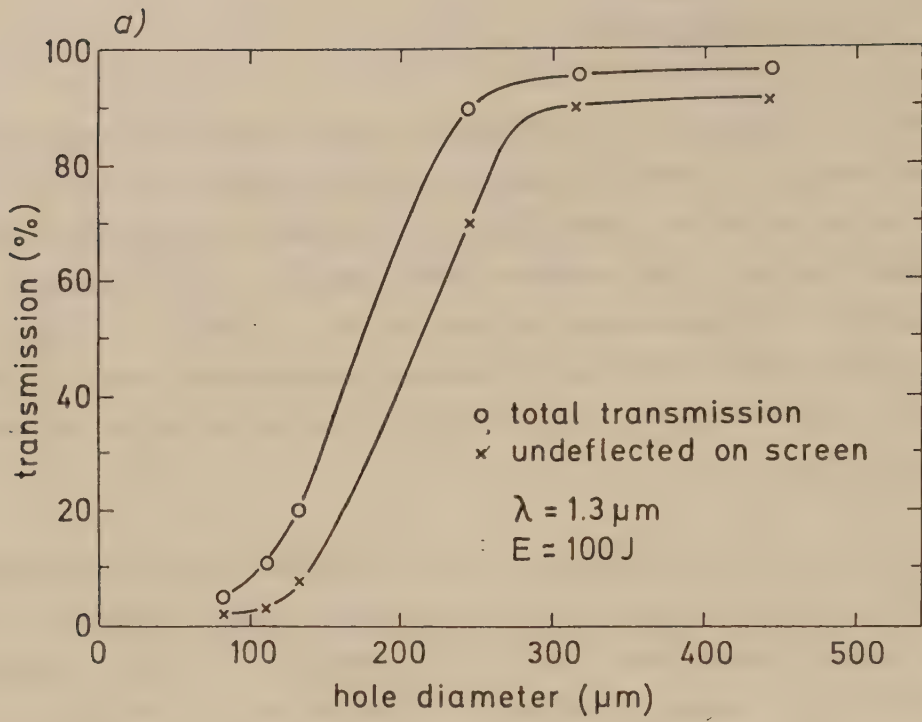


Fig. 21: Fraction of total transmitted and undeflected laser light as a function of the pinhole diameter.

a: For $\lambda = 1.3 \mu\text{m}$ (1 ω) and laser energy $E \cong 100 \text{ J}$

b: For $\lambda = 0.4 \mu\text{m}$ (3 ω) and laser energy $E \cong 30 \text{ J}$.

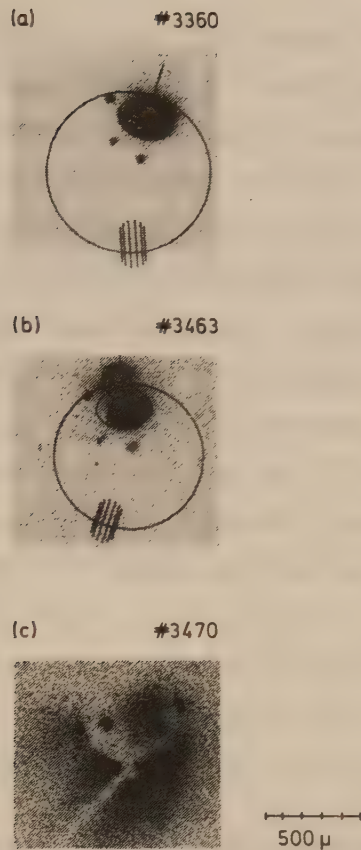


Fig. 22: Pinhole pictures taken from the front side of the cavities with the multipinhole camera. On Figs. a and b the contours of the cavity are drawn as seen from the camera.

the entrance hole of cavities. With cavities, the transmission can be further reduced due to plasma filling of the whole cavity enhancing the back-reflection of laser light and due to internal heating of the material surrounding the entrance hole.

With the aid of the pinhole camera different aspects of each shot were controlled, like the centering of the beam focus with respect to the entrance hole, the occurrence of laser double pulses and the target quality. In Fig. 22 we show pinhole pictures taken in three different shots. On Figs. 22.a and b we have added, the contour of the cavity as seen from the pinhole camera (the camera observed the front side of the

cavity from a direction 30° below the laser axis). In the first picture three satellite foci are seen which surround the left hand side of the entrance hole. These secondary foci were produced by beam splitters located between the laser and the focusing lens for beam diagnostic purposes. These foci were used as a reference for finding the location of the main focus in each shot. The first picture shows a satisfactory shot where the beam was correctly centered into the entrance hole. The second picture shows an example of a laser double pulse where the main pulse is followed a few nanoseconds later by a second pulse. In front of the entrance hole, a second radiation source is seen. It is attributed to material which has expanded through the entrance hole between the pulses and is reheated by the second pulse. On the third picture we can see how expanding plasma, escaping from a slit in the lower part of the cavity, generates a backlitting picture of the cavity. This target had been broken accidentally before the shot and the resulting cracks in the wall are responsible for the escape of hot plasma from its interior. In the experiments care was taken concerning the pulse and target qualities, avoiding double pulses or defective cavities, which may produce, aside of energy losses, confusing radiation sources.

Not only the pictures taken with the pinhole camera offer a space resolved image of the radiation source but spatial resolution is maintained in the direction perpendicular to the spectra taken with our spectrometer camera. Fig. 23.a shows the spectrum obtained from a big cavity with both diagnostic holes perfectly resolved. It was taken with a 1000 lines/mm, 25 μm diameter pinhole and a 1000 $\text{\AA}$ thick carbon filter. Fig. 23.b shows a characteristic spectrum from a small cavity taken with a 1000 lines/mm, 50 μm diameter pinhole. It is seen that the radiation with a wavelength shorter than 50 $\text{\AA}$ comes from the diagnostic holes, but beyond 50 $\text{\AA}$ the radiation comes from an extended source, even larger than the original cavity dimensions. A plausible explanation is, that after a few nanoseconds, when the cavity has expanded (due to the extremely high pressures in its interior) to many times of its original dimension, the wall becomes very thin and also transparent to the radiation still enclosed inside the cavity. This extended radiation source emits at longer wavelengths indicating cooling during expansion.

The large amount of information contained in these spectra and their small physical size (typically 100 μm in width) makes their analysis by means of conventional densitometers rather hopeless. It was therefore necessary to use more sophisticated methods of computer supported densitometry. This was performed with the facilities available at the European Southern Observatory. Each spectrum was read by a digital densitometer with a resolution of 100 μm in the direction parallel to the spectrum and 5 μm in the direction perpendicular to it. The film density in each pixel was then converted into energy density by means of the density and sensitivity curve shown in Fig. 20.a and b. As an example of the capabilities of the processing system, we show an isodensity-chart in Fig. 24.a and a three dimensional plot of the spectrum in Fig. 24.b.

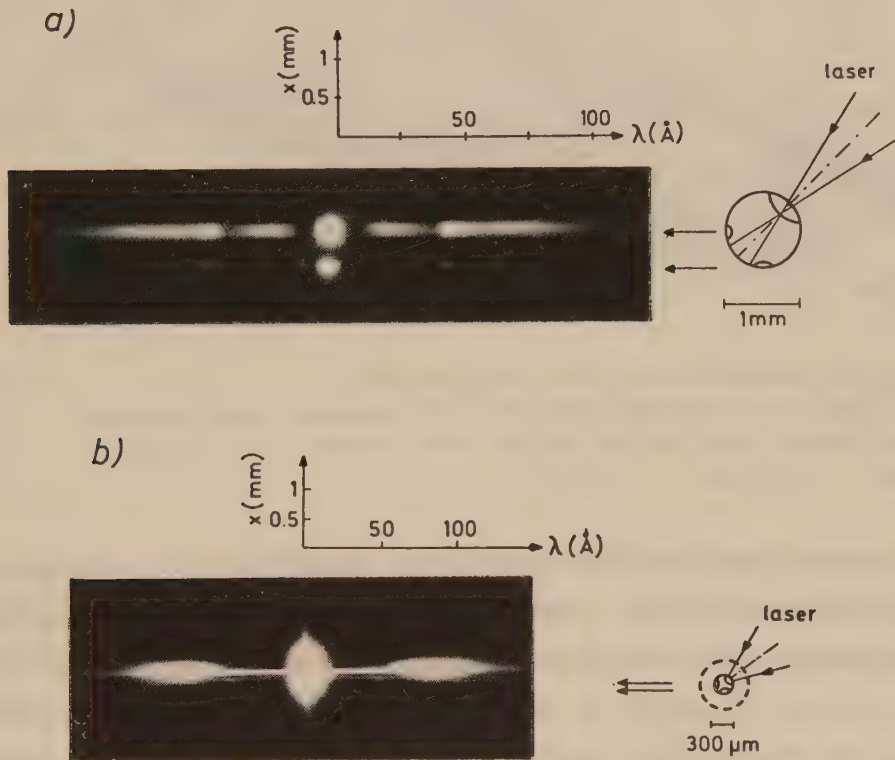


Fig. 23: Spectra obtained with the spectrometer on film.

a: from a big cavity, b: from a small cavity.

The relative position of cavity and the laser beam is also shown.

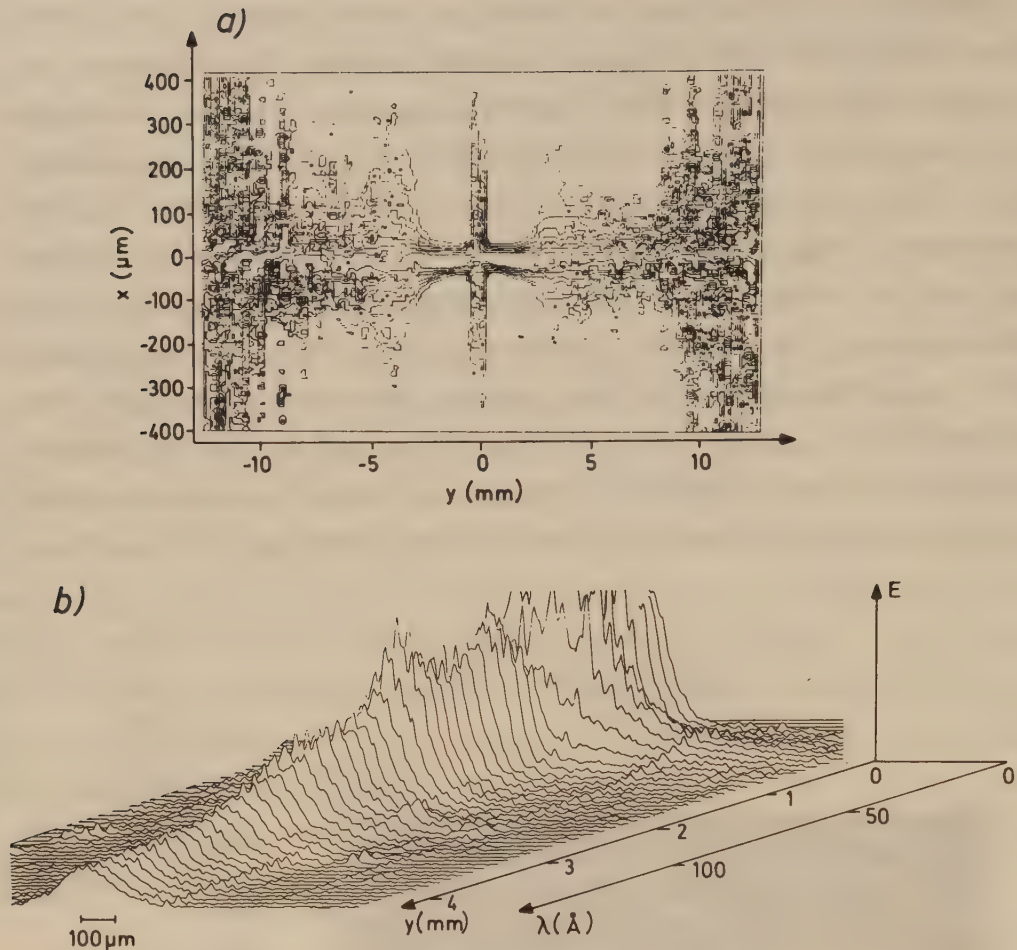


Fig. 24a: Isodensity-plot of a soft x-ray spectrum.

b: Three-dimensional plot of a soft x-ray spectrum. Both diagrams were obtained with the aid of the IHAP SYSTEM at the European Southern Observatory in Garching.

The transverse profile of the first order spectrum of the radiation from the diagnostic hole can be predicted by taking into account its diameter, the magnification and the pinhole diameter of the spectrometer. We have calculated it in Appendix 3 under the approximation that diffraction effects due to the pinhole size are negligible compared with the geometrical resolution. A comparison between the calculated and measured profiles is made in Fig. 25.a and b where we show scans (transverse to the dispersion direction) through the spectrum at different wavelengths. The only fitted parameter is the maximal intensity at the center of the scan. With this type of plots we are able to decide what portion of the incident radiation comes from the diagnostic hole, and which one from other sources; for example, as was mentioned above, when the small cavities radiate as a whole during a later stage of their expansion, they constitute a diffuse additional source of soft x-ray radiation.

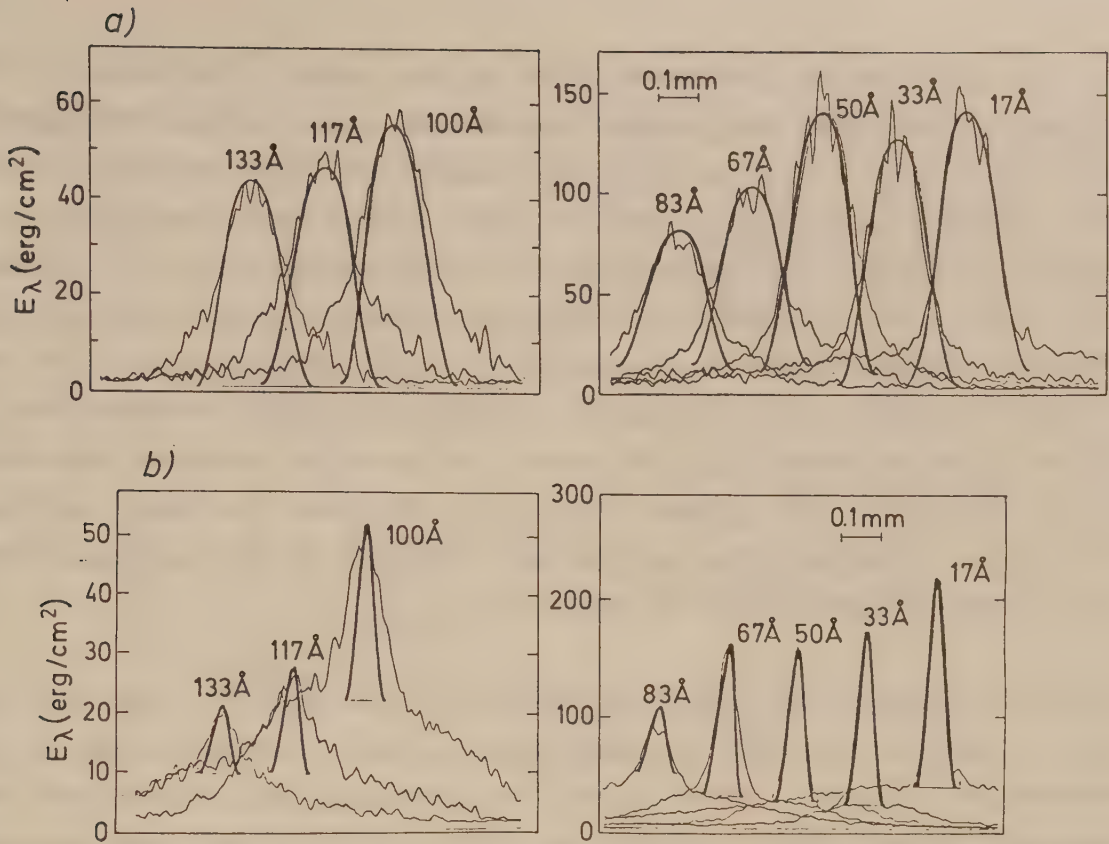


Fig. 25: Comparison between the measured film density scans perpendicular to the spectrum and the profiles calculated in Appendix 3 at different wavelength for a) a big cavity, b) a small cavity.

Fig. 25.a shows scans through the spectrum obtained from a 200 μm diameter diagnostic hole (large cavity) and Fig. 25.b shows scans through the spectrum obtained from a 50 μm diameter diagnostic hole (small cavity). In the first figure we see excellent agreement between the calculated profiles and the experimental scans, indicating that in this shot the radiation comes only from the diagnostic hole for the whole spectral range. In the second figure we also see excellent agreement up to a wavelength of 80 Å where diffraction effects at the pinhole become noticeable. We see also the intense background produced during expansion of the cavity at wavelengths larger than 50 Å.

The spectral intensity distribution emitted through the diagnostic hole may be obtained either from the energy density at the center of a scan

or from the integral performed over its full width. The details of the transformation from the energy density on the film to the spectral intensity of the source are given in Appendix 3, here we give only the final result in graphical form. In Figs. 26-28 we have plotted the spectral intensity I_λ , defined as the energy emitted per unit surface, unit time, unit wavelength intervall and per unit solid angle in the direction perpendicular to the plane of the diagnostic hole. For comparison we show also black-body spectral intensities corresponding to different temperatures. Fig. 26 shows the spectrum obtained from a big cavity irradiated with 1w laser light, Fig. 27 shows the spectrum from a small cavity irradiated also with 1w light and Fig. 28 the spectrum from a small cavity irradiated with 3w laser light.

As can be seen in Fig. 23a (big cavity) the zeroth order and the first order spectrum are well separated by a clear, unexposed gap in the wavelength range 0 - 15 Å as one would expect for a blackbody-like spectrum. Nevertheless, because the film sensitivity decreases strongly below 15 Å, it cannot be excluded that some radiation emitted in this range escapes detection with the type of film used here. For the small cavities (Fig. 23b), where the temperature is higher and the gap narrower, problems arise in addition due to the overlapping of the zeroth order with the first order. In both cases the spectra could therefore not be evaluated below 15 Å.

Our primary interest in this work is in the energy flux connected with the thermal radiation in the cavity rather than its exact spectral distribution. The energy flux is the quantity which appears explicitly in our theoretical treatment (which used frequency averaged transport coefficients) and may still be reasonably well predicted even if the spectrum deviates in some spectral regions from a Planck distribution. Therefore we introduce a wavelength integrated quantity B_E , the experimentally measured radiant emittance radiated towards the outside from the plane of the diagnostic hole. It is defined as the integral over the spectral intensity I_λ

$$B_E = \pi \int_\lambda I_\lambda d\lambda$$

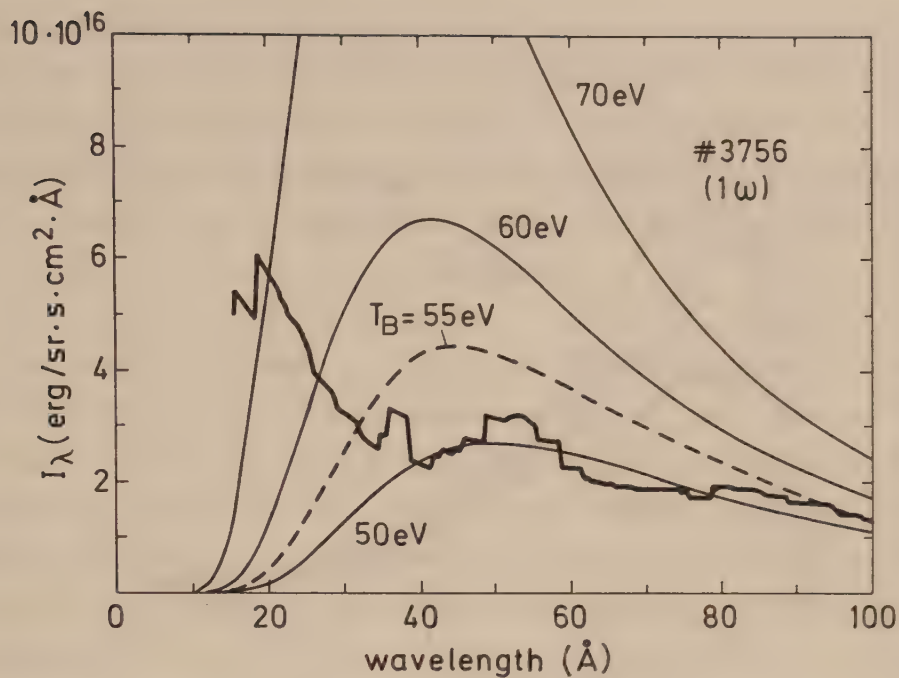


Fig. 26: Spectral intensity distribution I_λ (erg/sr.s.cm².Å) obtained from a big cavity irradiated with a 1.3 μm pulse. Black-body spectra for various temperatures (in eV) are shown for comparison.

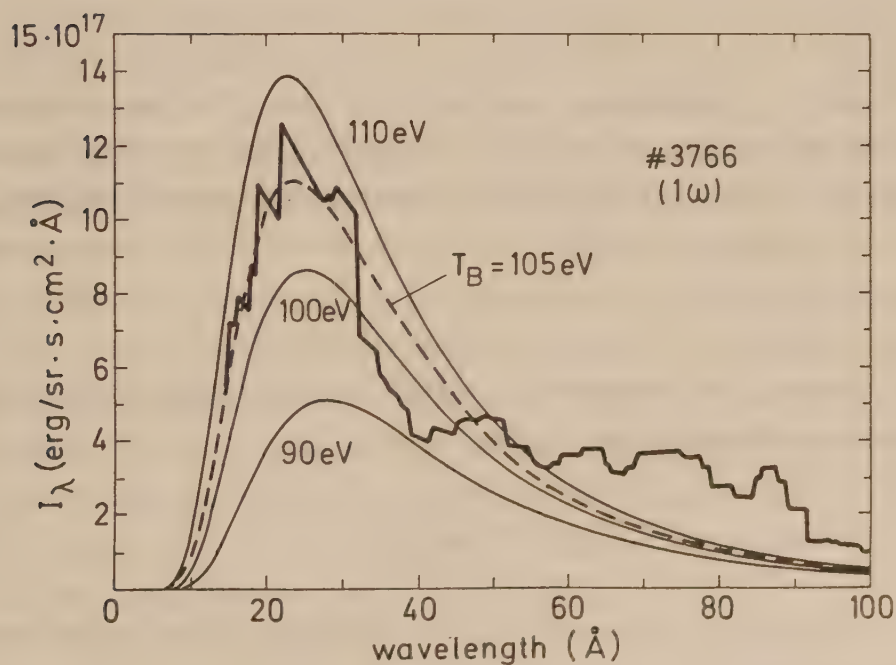


Fig. 27: Spectral intensity distribution I_λ obtained for a small cavity irradiated with 1w light. Black-body spectra are shown for comparison.

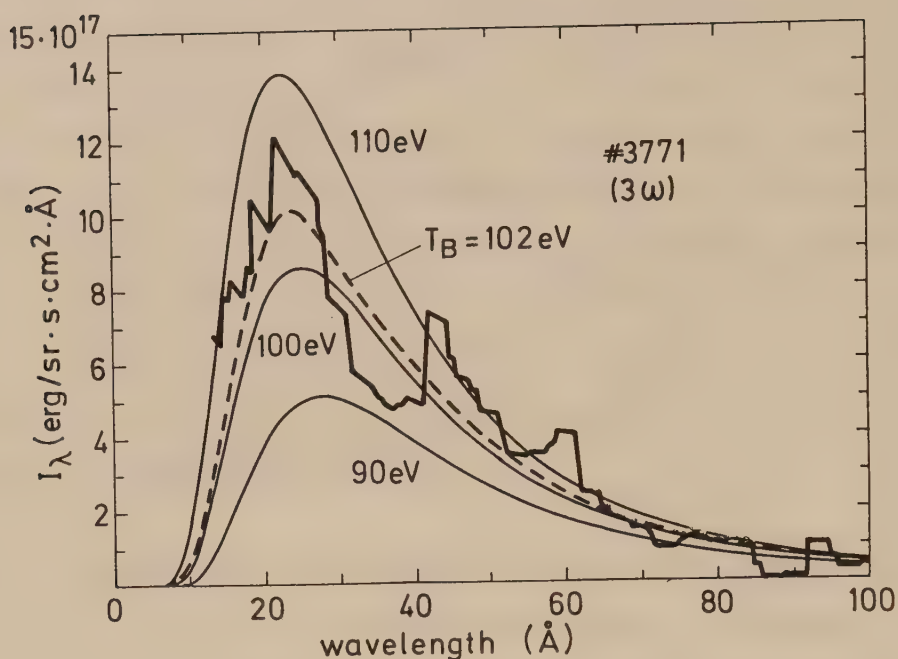


Fig. 28: Spectral intensity distribution I_λ obtained for a small cavity irradiated with 3ω light. Black-body spectra are shown for comparison.

The factor π is the result of an integration over the half space (this point is further discussed in App. 3). Over the experimentally measured spectral range of $15 \text{ \AA} \leq \lambda \leq 150 \text{ \AA}$ the wavelength integral contains more than 98 % of the radiated energy for Planck spectra with a temperature in the range 50-120 eV.

In order to compare the measured spectra with a Planck spectrum we define a brightness temperature T_B by

$$T_B = (B_E/\sigma)^{1/4}$$

The Planck distribution corresponding to the brightness temperature is marked in Figs. 26-28.

For the big cavities we find $T_B = 55 \text{ eV}$ for 1ω irradiation. For the small cavities we find $T_B = 105 \text{ eV}$ for the 1ω irradiation and $T_B = 102 \text{ eV}$ for

the 3w irradiation. Thus the small cavities attain a much higher temperature than the big ones due to their smaller inner surface area. The fact that the temperature is nearly equal for 1w and 3w irradiations of small cavities is probably due to the fact that the absorbed energy is about equal (as shown by recent absorption measurements, see Sec. VII).

If the spectral shape is compared with the Planck spectrum corresponding to the brightness temperature it is seen that deviations occur. For both cavities bands are observed in the range 20-30 Å and 40-50 Å (tentatively identified with the N and O bands, respectively, of multiply ionized gold /22,26/.) The deviations are largest for the big cavity near 20-30 Å. We attribute this excess radiation to the transition region between the hot (~ 1 keV) laser produced plasma in the center of the cavity and the wall where the absorbed laser energy is gradually transformed into radiation (for a more detailed discussion see Sec. VII). If we assume that the radiation produced in this transition region is of the order of the mean absorbed laser intensity, the emission of the wall (supposed to emit blackbody radiation) would be larger by a factor N (remember that $N = \text{blackbody intensity} / \text{source intensity}$). One would expect therefore that the relative importance of the radiation produced in this primary step of thermalization diminishes as N increases. In fact, if we consider the spectra of the small cavities (Fig. 27 and 28) where N is larger (see Sec. VII) we see that the approach to a Planck spectrum is much better. This observation is interpreted as the predicted approach to equilibrium conditions as the source intensity in the cavity is raised.

A detailed evaluation of the experimental results will be given in Sec. VII. Here we note only that the error in the measurement of I_λ is given by the uncertainty in the value of the film sensitivity (20 %), the uncertainty in the value of the grating efficiency in the first order and the contributions from higher orders (20 %), and by the subtraction of the underground (10 %). Together this gives an error of about 50 % in I_λ and the integral over the spectrum B_E .

VII Discussion

In this section we compare the experiments described in Sec. VI with the theory developed in Secs. III and IV. We start with a qualitative description of the main process occurring inside the cavity upon laser irradiation.

VII.1 Physical mechanisms of laser energy deposition in a cavity

The laser beam enters the cavity through the entrance hole (with the focal plane in the plane of the entrance hole in order to minimize the hole diameter and hence losses from the interior of the cavity), propagates through the cavity and interacts with matter when it hits the rear wall of the cavity. Here the process of thermalization of the laser energy begins. It can occur through a variety of intermediate steps which even for the more accessible geometry of a planar target are far from being understood in a quantitative manner /19/.

Due to heating by laser light a laser-produced plasma forms on the rear wall of the cavity. The absorption of laser light in the "hot spot" on the rear wall is, however, not complete. Assuming that absorption is similar as on planar targets /19/ one expects for 1ω irradiations 15-30 % and 30-50 % absorption, for small and large cavities, respectively. For 3ω irradiations the corresponding values are 30-50 % and 40-60 %, respectively. A large part of the laser energy is thus reflected from the hot spot on the rear wall and, in general after several reflections, deposited on the other parts of the inner wall.

The absorbed laser light (in the hot spot or elsewhere in the cavity) is absorbed by the electrons in the plasma near the so-called critical density $n_c = (\epsilon_0 m_e \omega^2 / e^2)$. In the relatively low-density region accessible to the laser light the temperature is rather high (typically ~ 1 keV, depending on laser wavelength and intensity) and the plasma is optically thin for its thermal radiation. This plasma is therefore not directly the source of black-body radiation in the cavity.

The extremely high temperature gradient between the laser heated plasma and the wall causes a heat flow due to electron heat conduction

towards the wall. As the temperature drops near the wall, the density rises and the heated layers are able to radiate more efficiently thermal radiation than the optically thin laser heated plasma. In this manner, part of the absorbed energy is distributed inside the cavity as plasma thermal radiation. Rather complex atomic physics processes occur in this transition layer where this primary conversion from laser light into soft x-rays occurs, no simple models for its description exist, and one must appeal to multigroup computer calculations /26/.

It should be noted that the transformation of laser light into soft x-rays is most desirable for the heating of the cavity, because it allows to heat an empty cavity in a more uniform way. Short wavelength lasers are advantageous in this respect because the conversion efficiency is better and the soft x-ray spectrum softer (because the laser light is absorbed by a plasma layer of higher density and lower temperature).

The fraction of absorbed laser energy not transported away by radiation is transformed into internal and kinetic energy of the heated material. Estimates show that due to the high temperature of the laser heated plasma and its fast expansion velocity the cavity can fill up to the order of the critical density with expanding plasma before the end of the laser pulse (at least for 1 ω irradiations of the small cavities) and thus energy can also be transported across the cavity by electron thermal conduction and convection. Note that in a (fully ionized) plasma the heat conductivity coefficient is proportional to $T^{5/2}$ but independent of the density.

The so-called suprathermal electrons may also be of importance for the transport of energy inside the cavity. They are generated by linear (resonance absorption) and nonlinear (parametric instabilities) absorption mechanisms, especially at high intensity irradiations with long wavelength laser light. For 1 ω irradiations of a small cavity, with a laser intensity of 10^{13} - 10^{14} Wcm⁻² on the rear wall, measurements with planar gold targets suggest a "hot" temperature of these electrons of 5-10 keV /19, 23,26/. In the presence of such fast electrons, fast ions are generated in the ambipolar electric field of the laser produced plasma, enhancing the material propagation. As the cavity fills up with plasma, the ambipolar field disappears and the electrons can then transfer energy freely through the cavity.

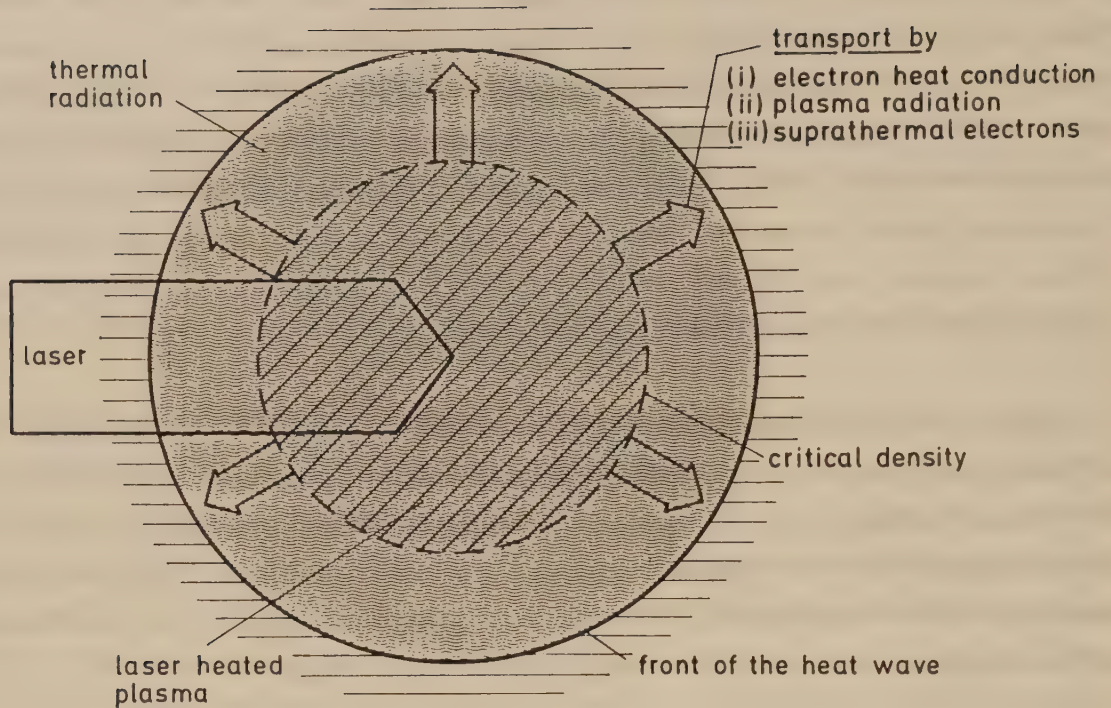


Fig. 29: Model for the laser heating of a cavity. The laser energy is coupled into a low-density plasma source located at the center of the cavity. The energy transfer from the source to the wall can be accomplished by electron heat conduction, plasma radiation or suprathermal electrons.

The relative importance of these transport mechanisms depends on irradiation wavelength and intensity and on geometry and cavity size. Due to the complexity of the phenomena a quantitative treatment is practically impossible. This raises the question whether our theoretical results can be applied to such a complex situation.

VII.2 The generation of thermal radiation in a laser heated cavity

It is often the case in physics that a thermalization process is too complex to be understood in detail; nevertheless such a process may lead to a simple and well defined state. In our problem the expectation is that during thermalization the absorbed laser energy (or at least an appreciable part of it) passes through the state of blackbody-like thermal radiation, characterized by the single parameter T .

Table VIII.
Energy transport coefficients

i) electron heat conduction

$$\kappa = \frac{3\gamma_0(z) k(kT)^{5/2}}{4\sqrt{2}\pi\Lambda \sqrt{m_e} Ze^4} \text{ erg/s cm } ^\circ\text{K}$$

$$n = 8.05 \times 10^{-12} \text{ g s cm}^{-3}$$

$$v = 2; \mu = 0; \lambda = 1$$

$\gamma_0(z) \cong 10$ (for $z \cong 20$); $\Lambda = 5$ (Coulomb logarithmus);
 m_e : electron mass; e : electron charge; n_i : ion density

Reference: /25/

ii) plasma x-ray radiation

$$\text{range: } \ell_v = 3.2 \times 10^{-3} E^3 \rho^{-1} \text{ cm}$$

$$E: \text{ photon energy in keV, } \rho: \text{ matter density in g cm}^{-3}$$

$$\ell_o = 3.2 \times 10^{-3} E^3 \text{ g cm}^{-2}$$

$$s = 0 \quad r = 1$$

Reference: See appendix 4

iii) suprathermal electrons

$$\text{residual range: } \ell_E = 1.9 \times 10^{-5} E^{1.6} \rho^{-1} \text{ cm}$$

$$E: \text{ electron energy in keV; } \rho: \text{ matter density in g cm}^{-3}$$

References: /23,24/

$$\ell_o = 1.9 \times 10^{-5} E^{1.6} \text{ g cm}^{-2}$$

$$s = 0 \quad r = 1$$

This may not occur if the laser heated plasma remains isolated from the wall and no radiation is generated or if during laser-plasma interaction a large fraction of the absorbed energy is transformed into penetrating radiation (photons or particles) which is absorbed at a great depth in the wall. In the latter case the calculated wall temperature would not be reached. In this section we examine these possibilities, making use of the formalism developped in Section IV. We simplify the complex phenomena in the cavity by the model shown in Fig. 29. In this model the laser is assumed to generate a hot, low density plasma bounded towards

the more dense zones near the wall by the critical density. This laser-heated plasma is considered as the source of a flow of energy directed towards the wall. We assume a spherical symmetric situation as expected and supported by high-speed photography of the explosion of the thin-walled, small cavities taken by Maaswinkel and collaborators in our laboratory.

In the following we shall consider three possibilities for the transport of energy from the laser heated plasma to the wall and examine its consequences for the generation of radiation: (i) electron thermal conduction, (ii) radiative transport by (monochromatic) x-ray radiation, and (iii) energy transport by suprathermal electrons. Depending on the case we specify either the coefficient of heat conduction $\kappa = \kappa_0 T^\nu v^\mu$ or the mean free path $\ell_s = \ell_0 W^r v^s$ of the source radiation. The expressions which we have used are given in Table VIII for the three cases.

We consider first the case (i) of electron thermal conduction. Similar to our treatment of wall heating by radiative conduction we suppose that an electron heat bath is brought into contact with the wall at $t = 0$. With respect to our theoretical analysis, electron heat conduction and radiative conduction differ in their constants and exponents, but in principle all the regimes discussed in Sec. IV have their counter-part for electronic heat conduction. They are shown together with the radiative regimes and the various border lines in Fig. 30.

Let us follow a line of constant energy source flux, say $S_0 = 10^{19} \text{ erg s}^{-1} \text{ cm}^{-2}$. First an electron heat wave forms, which, after $4 \times 10^{-13} \text{ s}$, develops into an electron ablative heat wave. If we ignore for a moment the radiative regimes (i.e. if electron heat conduction is the only transport mechanism) the temperature would rise towards the right following the line of constant S_0 of the electron AHW regime. This line is therefore continued as a dashed line on the right hand side of the radiation NEQ-AHW regime. It is seen that after 10^{-9} s a temperature of about $10^7 \text{ }^\circ\text{K}$ is reached.

There is little doubt that such a high temperature is indeed achieved in the low-density plasma where the laser radiation is absorbed. This

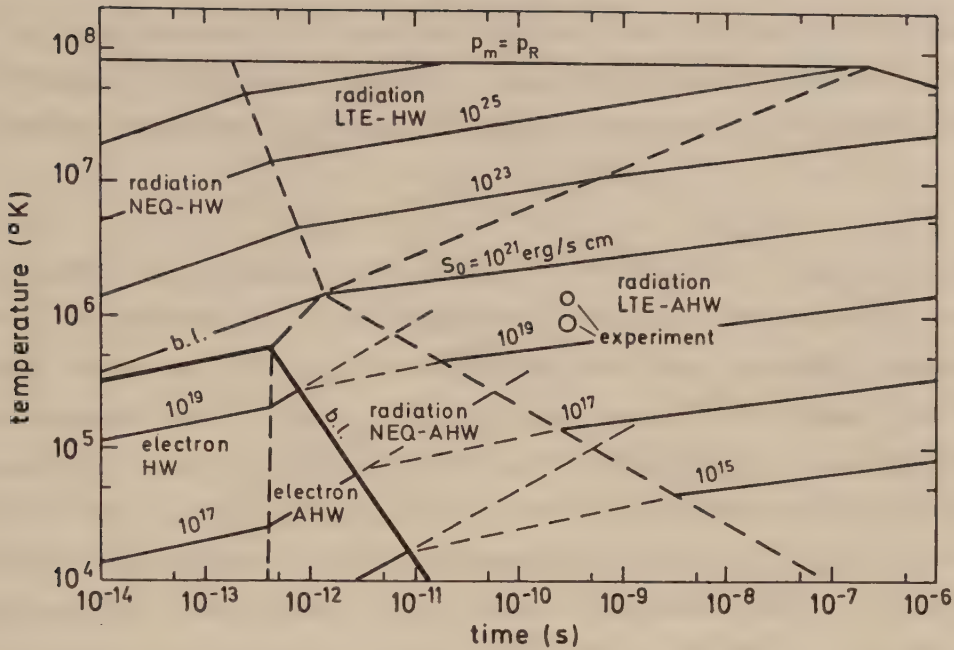


Fig. 30: Border lines and lines of constant source flux S_0 for the self-similar regimes of radiation and electron heat conduction. On the border lines labeled b. l. the depth of the heated material and the velocity of the thermal wave are equal for both processes. In the regions labeled electron-HW and electron-AHW the heated mass and wave velocity of the electron regimes are larger than those of the radiation regimes. In the other regions the opposite holds. For sufficiently small temperatures ($T < 5 \times 10^5$ K) and short times ($t < 5 \times 10^{-12}$ s) the heating process is dominated by electron thermal waves.

plasma is optically thin for its own thermal radiation, radiative transport is unimportant and electron heat conduction constitutes the only means to transport the absorbed laser energy away (we ignore here the possibility of transport by suprathermal electrons). The very high temperature (compared to the temperatures achieved in the presence of radiative heat conduction) is a consequence of the fact that electron heat conduction is far less effective in this temperature range than radiative conduction.

From the directly heated plasma the heat diffuses towards the cold wall into zones of lower temperature and higher density i.e. of increasing opacity. One would expect therefore that a radiation driven wave forms ahead of the electron driven wave which, as more mass is heated with

time, dominates the character of the wave; in this way a transition to the radiative regimes may occur. It turns out, however, that this transition cannot be treated by the present method. The reason is that obviously both waves exist simultaneously, though in different regions of space; our treatment is, however, based on spatially averaged quantities only.

Nevertheless, it may be useful to illustrate the transition problem by a few remarks related to Fig. 30. We remember that the radiation NEQ-AHW regime assumes a source with the property $\ell_s = \ell_R$ i.e. a wave is produced with an optical thickness equal to one. One of the reasons why the concept of such a (fictitious) source is useful is that it allows to judge the optical thickness of a wave. If, at a given time, a source has produced a wave with a lower temperature this wave will be optically thick (because the wave depth is larger and because with our assumptions the opacity increases with decreasing temperature), and vice-versa. In this way it is seen from Fig. 30 that the electron HW and the electron AHW are optically thick until the border line b.l. where the radiation NEQ-AHW regime is reached. On this border line the optical thickness of the wave is equal to one. If electron heat conduction determines the wave also on the right hand side of this border line its optical thickness becomes less than one. Estimates have shown that it cannot be excluded that the fraction of the absorbed source flux radiated by the wave goes to zero with time and thus a radiation driven wave cannot develop. Should this occur only a very high-temperature, optically thin plasma would form in front of the wall storing the absorbed energy as internal and kinetic energy. This problem certainly deserves further study and clarification. On theoretical grounds we cannot decide whether the coupling to the radiation driven wave occurs or not; the answer must be left to the experiments. They show, as we shall show in more detail in Sec. VII.3 that this coupling indeed occurs.

Let us discuss now the possibility that the absorbed energy is lost deep into the wall by "hard" plasma radiation produced in the transition region between laser-heated plasma and wall (case ii) or by suprathermal electrons generated in the laser absorption region (case iii). Both cases can be treated along the same lines if we assume that the mean free path of the photons or electrons is determined by two-body collisions and

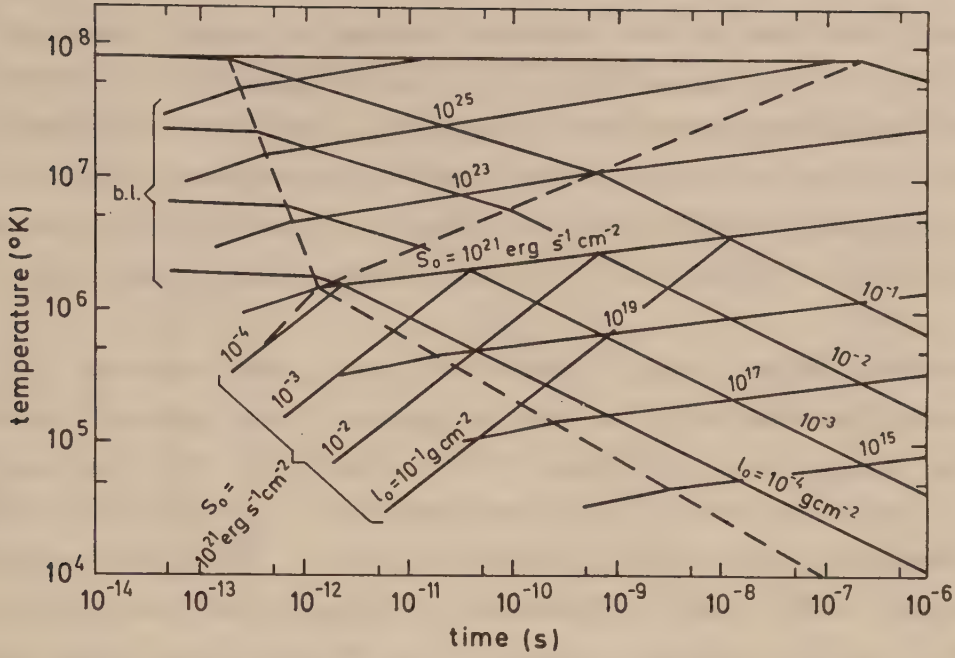


Fig. 31: Border lines (b.l.) and lines of constant source flux S_0 between the NEQ regime (lower left) of source radiation with a temperature independent range l_0 and the radiation regimes (upper right). For clarity the lines of constant S_0 are drawn only for $S_0 = 10^{21} \text{ erg s}^{-1} \text{ cm}^{-2}$. The lines for the radiation regimes are from Fig. 10. The circles represent measured temperatures for large (large circle) and small cavities.

does not depend on the temperature of the heated material. In this case we have

$$\ell_s = \ell_0 v ,$$

where the exponents r and s are now $s = 0$ and $r = 1$. The coefficient ℓ_0 (equal to the range in g cm^{-2}) depends on the energy of the photons or electrons.

For this particular case the NEQ regimes 1 and 3 are not distinguished because $A_1 = A_3 = \ell_o$. The single NEQ regime is shown in Fig. 31 together with the lines of the radiative regimes as shown previously in Fig. 10 (with the N_o lines omitted). It is again located in the lower left part of the figure, separated from the radiative regimes by a border line b.l. Several border lines are drawn for four different values of the parameter ℓ_o . For simplicity lines of constant S_o are given in the NEQ regime only for $S_o = 10^{21} \text{ erg s}^{-1} \text{ cm}^{-2}$, but for the four values of ℓ_o .

Let us also here follow a line of constant S_o for a given value of ℓ_o . For the case under consideration the source heats a fixed amount of matter, the internal energy W rises linearly with time and, for the assumed equation of state $W = KT^{3/2}$, the temperature in proportion to $t^{2/3}$. When the border line b.l. with the same value of ℓ_o is reached, radiation heat conduction sets in and the temperature follows from now on the line of constant S_o of the radiative regime. From the arguments given above it is seen that in this case (for $\ell_o \geq 10^{-4} \text{ g cm}^{-2}$) the wave is all the time optically thick for its own thermal radiation and the transition to the radiation regime is not problematic.

Once the border line has been reached for a given S_o and ℓ_o , the temperature is determined by radiative heat conduction and depends no longer on the range of the source radiation. In terms of Fig. 2 the crossing of the border line corresponds to a transition from case (a) to (b) and later an asymptotic approach to the ideal case (c). From the position of the experimental points shown in Fig. 31 it is seen that the transition from (a) to (b) occurs before the end of the laser pulse if $\ell_o \leq 2 \times 10^{-3} \text{ g cm}^{-2}$ (large cavity) resp. $\ell_o \leq 5 \times 10^{-3} \text{ g cm}^{-2}$ (small cavity). We examine now whether these conditions are fulfilled for the cases (ii) and (iii). In the case (ii) where the absorbed energy is assumed to flow into the wall as "hard" x-ray radiation a certain difficulty arises because its range has a temperature dependence which is not readily expressed in the form of a power law. We consider therefore only two limiting cases, cold gold and gold heated to a temperature of 100 eV.

The photon range in cold gold is shown in Fig. 32. It is seen that the tolerable range of $2 - 5 \times 10^{-3} \text{ g cm}^{-2}$ is exceeded only for photons with

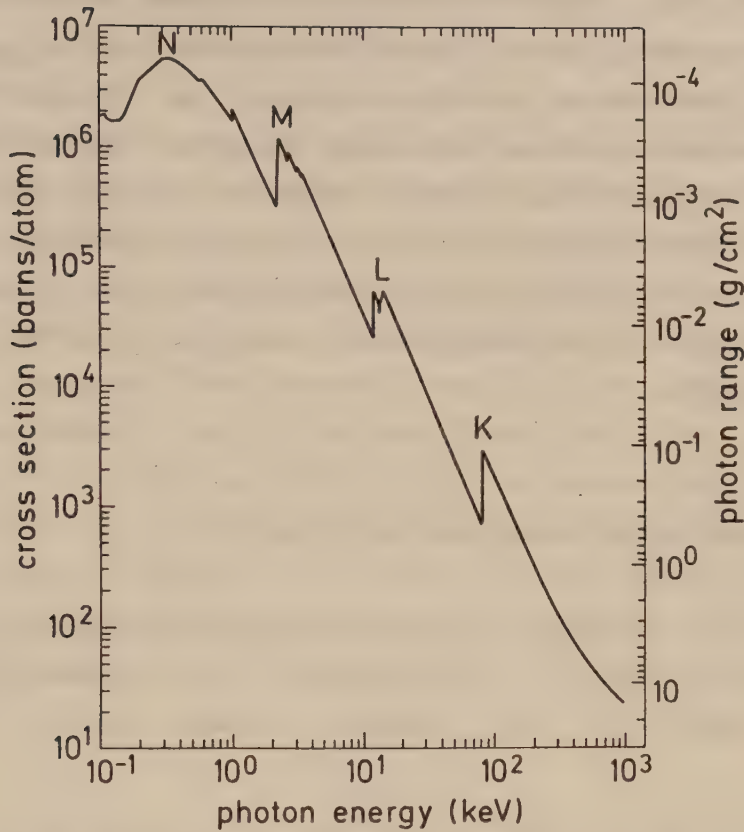


Fig. 32: Photon range in cold gold. After Ref. /27/.

an energy in excess of several keV. The range of photons in gold with a temperature of 100 eV reaches $3 \times 10^{-3} \text{ g cm}^{-2}$ for a photon energy of 1 keV (see Table VIII). Experimentally we measured the spectrum radiated by the cavities for $h\nu < 800 \text{ eV}$ and found the shortest wavelength peak near $\sim 600 \text{ eV}$. Obviously photons with such relatively low energies fall into the tolerable range.

For case (iii) with suprathermal energy transport into the wall we use the residual range, measured along the electron trajectory, to describe the penetration of the source radiation. This is a pessimistic approach because most of the energy is deposited at a depth of $\ell_0/\sqrt{27}$, see Ref.

/23/. Using the convenient range formula /23/ given in Table VIII it is seen that the tolerable range corresponds to fast electrons with an energy of 18-33 keV. This is well beyond the fast electron energies of 5-10 keV corresponding to our experimental conditions (see above).

We can state the results of this discussion as follows: If we make reasonable assumptions about the main transfer mechanisms of laser energy into black-body-like thermal radiation, it is found that for our experiments thermalization of the laser energy should result in a radiation field in the cavity with a temperature close to that predicted for an ideal source. This expectation seems to be confirmed by the experimental results.

VII.3 Comparison of experiment and theory

We proceed now to compare the experimental data with the theoretical predictions made in Sec. III. As discussed in the preceding section the appropriate regime is the radiation driven ablative heat wave (LTE-AHW).

The basic measured parameter of the cavity is the experimental radiant emittance B_E as defined previously in Sec. VI:

$$B_E = \pi \int_{\lambda} I_{\lambda} d\lambda$$

From the measured value of B_E we derive an experimental wall temperature T_E in analogy with eq. (53.a) by the relation

$$B_E = \sigma T_E^4 [1 - \exp(-N_E)]$$

where $N_E = \sigma T_E^4 / S_s$ is the experimental quality factor. S_s is the effective energy flux arriving at the wall of the cavity, defined as:

$$S_s = \eta_{abs} E_L / 4\pi R^2 \tau \quad ,$$

where η_{abs} is the fractional absorption of laser energy inside the cavity, E_L the laser energy, R the cavity radius, and τ the temporal width of the laser pulse (taken as 3×10^{-10} s). Note that S_s corresponds to the quantity S_v used in the theoretical sections.

We note that during the writing of this work new absorption measurements were made by Földes and colleagues in our group by surrounding the cavities with a newly constructed, calibrated Ulbricht sphere. The values of η_{abs} shown in Table IX were taken from these measurements. It is seen that the cavity targets absorb not as much energy as is transmitted through holes (corresponding to the entrance hole size) in planar targets (see measurements in Sec. VI, Figs. 21a and 21b). This is not unexpected because with a cavity target the wall around the entrance hole is also heated from the inner side, thus enhancing reflection and possibly closure of the hole for laser light due to plasma filling. The absorption measurements show the same trend as our transmission measurements, with fractional absorption being lowest for 1w irradiations of small cavities (which have the smallest entrance hole). It amounts to only $\eta_{\text{abs}} \cong 0.3$ in this case (the transmission measurements of Fig. 21 would suggest $\cong 0.5$). For the large cavities and 3w irradiations η_{abs} approaches one.

Table IX
Laser energy E_L , cavity inner area a_c , percentual hole transmission η_{abs} ;
energy flux S_s , energy reflexions in the cavity N ,
emitted radiant energy flux B and temperature T for three
characteristic shots.

shot	E_L (J)	a_c (cm ²)	η_{abs} (%)	S_s (erg cm ⁻² s ⁻¹)	N_E	N_T	B_E (erg cm ⁻² s ⁻¹)	B_T (erg cm ⁻² s ⁻¹)	T_E (eV)	T_T (eV)
# 3756	77 (1w)	4.00×10^{-2}	80	5.13×10^{19}	0.4	0.5	8×10^{18}	9×10^{18}	68	69
# 3766	69 (1w)	2.41×10^{-3}	25	2.39×10^{20}	0.8	0.7	11×10^{19}	8×10^{19}	117	111
# 3771	27 (3w)	2.72×10^{-3}	85	2.81×10^{20}	0.7	0.7	9×10^{19}	9×10^{19}	115	115

Table X

Measured parameters for the shots analyzed in Table IX

shot	cavity diam. (μm)	entrance hole diam. (μm)	diagn. hole diam. (μm)	wall thickness (μm)
# 3756	1.128	309	198	~ 30
# 3766	277	172	53	~ 2
# 3771	294	151	60	~ 2

Table XI

Calculated values for the ablated mass m_f , ablation depth ℓ_f ,
plasma density at the sound point $\rho_{M=1}$ and pressure on the wall P
for three characteristic shots.

shot	$m_F(\text{gcm}^{-2})$	$\ell_f(\mu\text{m})$	$\rho_{M=1}(\text{gcm}^{-3})$	P(Mbar)
# 3756	5.8×10^{-4}	0.3	0.75 (= 4 % ρ_s)	6.5
# 3766	1.3×10^{-3}	0.7	1.2 (= 6 % ρ_s)	21
# 3771	1.5×10^{-3}	0.8	1.3 (= 7 % ρ_s)	24

In Table IX we give the laser energy E_L , absorbed laser flux S_s , radiated energy flux B_E , and experimental wall temperature T_E . These values are taken from the three shots whose parameters are listed in table X.

We now compare the measured quantities with the theoretical predictions of Sec. III. We obtained for the theoretical cavity temperature T_T as a function of the energy flux S_s and time t (from Tables II and III)

$$T_T = (K^{-2/3}/\tilde{S}_s^{4/13}) a^{-4/39} S_s^{4/13} t^{4/39} = 96 S_s^{4/13} t^{4/39} \text{ eV}$$

where S_s is measured in units of 10^{13} Wcm^{-2} and t in ns.

From T_T we calculate the theoretical radiant emittance B_T as

$$B_T = \sigma T_T^4 [1 - \exp(-\sigma T_T^4 / S_s)]$$

and the theoretical quality factor N_T as

$$N_T = \sigma T_T^4 / S_s$$

These parameters are also shown in table IX.

For completeness and in order to illustrate the conditions achieved in the cavity we calculate also a set of other quantities, such as the ablation depth ℓ_f , the plasma density $\rho_{M=1}$ at the sonic point and the pressure at the wall p_f . They are given by

$$\ell_f = m_f / \rho_s = (\xi_f / \tilde{S}_s^{7/13}) a^{2/13} S_s^{7/13} t^{11/13} / \rho_s = 1.2 S_s^{7/13} t^{11/13} \mu\text{m}$$

$$\rho_{M=1} = (\tilde{R}_{M=1} / \tilde{S}_s^{4/13}) a^{3/13} S_s^{4/13} t^{-3/13} = 0.7 S_s^{4/13} t^{-3/13} \text{ g cm}^{-3}$$

$$p_f = (\tilde{P}_f / \tilde{S}_s^{10/13}) a^{1/13} S_s^{10/13} t^{-1/13} = 9.9 S_s^{10/13} t^{-1/13} \text{ Mbar}$$

Again S_s is measured in units of 10^{13} Wcm^{-2} and t in ns. Values for these quantities corresponding to the experimental conditions are listed in Table XI.

Table IX shows that the agreement between the predicted and experimentally measured quantities is surprisingly good. The experimental

points were plotted previously in Figs. 30-31 in the standard T-t plane. As noted already they fall into the LTE-AHW regime.

For a critical assessment of the meaning and accuracy of the comparison experiment-theory let us come back first to the principal assumptions in the evaluation of the experimental results.

The theoretical model assumes that all the absorbed laser energy thermalizes inside the cavity and feeds a radiation driven thermal wave in the wall, that no other energy reservoir exists and that the cavity remains empty during the laser pulse (convergence effects of the flow were neglected).

These three assumptions have the consequence that the duration of the heating period is equal to the laser pulse duration and moreover, due to the continuing diffusion losses into the wall, the cooling of the cavity occurs very rapidly after the end of the laser pulse. Therefore we have made the additional assumption that the emission time is equal to the laser pulse duration. This assumption affects the evaluation of the radiated energy flux and the temperature (the latter varies, however, only with the fourth root of the emission time).

Our time-integrated measurements give no information about the processes occurring in the interior of the cavity and we cannot exclude that the situation is more complicated than envisaged in the model, despite the good agreement between the measured and calculated radiated energy. The point is that from a cavity (which confines not only radiation but also the kinetic and internal energy of the heated material) always about the same amount of radiation could be emitted, even if the steps of thermalization are different. For example laser energy could be absorbed initially in the transient "reservoir" of a hot, optically thin plasma which is decoupled from the wall (this possibility was discussed in the preceding section). This plasma would expand rapidly into the cavity, the convergence of the flow towards the center would lead to a reconversion of kinetic into thermal energy, increase the density and improve the possibilities for the generation of radiation. A closer link between experiment and theory needs additional time-resolved measurements as they are presently made by P. Herrmann and G. Tsakiris in our laboratory.

Uncertainties arise also from the limited accuracy by which the constants are known which enter into the calculation of the theoretical values. These are in particular the heat capacity of the wall material, represented by the constant K , and the heat diffusion coefficient represented by the constant a .

The accuracy of the constant K is related to the problem of the correct equation of state for the wall material. We find it difficult to address this problem in a quantitative manner within the frame of this work. However, in Appendix 2 it is shown that the value adopted in this work is in good agreement with the sophisticated SESAME library.

Our numerical value of the constant a is based on the Rosseland mean free path as given in Ref. /2/ for free-free and free-bound transitions. It is probable that bound-bound transitions contribute to absorption in the case of gold, reducing the mean free path and hence the diffusion losses into the wall. In this respect our calculations are pessimistic ones, possibly underestimating the attainable temperature in the cavity.

We can make use of the Maximum Opacity Theorem /14/ to estimate the maximum effect of bound-bound transitions on the results. The Maximum Opacity Theorem gives a minimal radiation mean free path $\ell_R^{\min}$ as

$$\ell_R^{\min} = A_\ell T^j \nu^{\mu'}$$

with $A_\ell = 3.5 \times 10^{-11} \text{ cm}^{-2} \text{ g}^{-1} \text{ }^\circ\text{K}^{-1}$ and $j = \mu' = 1$. With $\nu = j+3 = 4$ and $\delta = 3/2$ one obtains $a = (16/3)\sigma A_\ell / \delta K^{(\nu+1)/\delta} = 8.8 \times 10^{-28} \text{ cm}^{-26/3} \text{ g}^{2/3}$. For N , the quantity which is most sensitive to the radiation mean free path, one obtains

$$N_T^{\max} = \sigma K^{-8/3} \tilde{S}_s^{-16/13} a^{-16/26} S_s^{3/13} t^{16/26} = 10 S_s^{3/13} t^{16/26}$$

where S_s and t are again in units of 10^{13} Wcm^{-2} and 10^{-9} s , respectively. With $S_s = 2.5 \cdot 10^{13} \text{ Wcm}^{-2}$ and $t = 300 \text{ ps}$ (corresponding approximately to the conditions achieved in the small cavity) one obtains $N_T^{\max} = 6$ instead of $N_T = 0.65$. Thus bound-bound transitions could enhance N by a factor up to about 5.

One may ask why due to bound-bound transitions not a somewhat higher value of N was achieved in the experiment than theoretically predicted. In this context it is important to note that the measurement of total absorbed energy gives only an upper limit for the energy absorbed in the interior of the cavity and hence a lower limit for N_E . It is possible that a fraction of the energy was absorbed on the outer surface of the cavity and did not contribute to the heating of its interior. In addition we have neglected other processes connected with energy losses, like shock heating of the wall, particle or radiative preheat and convective or conductive losses through the holes. It is possible that the effect of such losses (not known precisely at present) is compensated by an enhanced opacity due to bound-bound transitions.

It is clear from this discussion that the good agreement between experiment and theory may be somewhat fortitious and more detailed investigations, both experimentally and theoretically, are needed. But future refinements of the investigations could change the situation in the sense that less energy is needed for the generation of the radiation field than estimated in this work.

VIII Summary and conclusions

At the beginning of this work we were faced with the open question whether the generation of a black-body-like radiation field inside a cavity with a temperature exceeding 10^6K would be possible by means of a laboratory pulsed power source, in particular a high power laser.

Using the concept of an ideal source it became clear that the temperature in the cavity is determined by diffusion losses of radiation into the wall. After a transient initial phase (of negligibly short duration for a powerful source) the radiation will approach thermal equilibrium with matter, the losses into the wall caused by heat diffusion decrease continuously and the radiation field becomes more and more isolated from the cold material in the deeper layers of the wall. Nonuniformities in the deposition of source energy are radiatively smoothed out as the intensity of the thermal radiation field and the number of energy "reflections" in the cavity increase and the conditions inside the cavity approach those of an ideal black-body radiation field. In this later stage the problem is entirely solvable inside the frame of the LTE (local thermal equilibrium) approximation.

In the theoretical part of this work we have shown that the general problem of wall heating can be treated as a sequence of different types of self-similar solutions. In this context we have analyzed and solved analytically for the first time the problem of an ablative heat wave driven by nonlinear heat conduction. The solution we have found is most important from a practical point of view because its range of validity covers the parameter range accessible with pulsed power sources available today in the laboratory.

Experimentally the ideal case where the energy delivered into the cavity is reflected many times before it is finally absorbed by the wall could not be reached due to the relatively modest power source available. However, temperatures slightly over 10^6K were attained in the laboratory using a pulse laser. It was experimentally shown that with increasing temperature the spectrum of the emitted radiation tends towards the blackbody limit and the comparison between theory and experiments shows good agreement.

The theory predicts a gradual approximation to an ideal blackbody radiation field inside the cavity for more powerful energy sources; it remains to be seen whether this prediction will be confirmed in the future with the use of more powerful laser or particle beams.

Appendix I. Total energy.

The expression for the total thermal energy at time t is

$$\begin{aligned} E_T(t) &= \int_{m=0}^{m_f(t)} \frac{p(m,t)v(m,t)}{\gamma-1} dm \\ &= \frac{p_1(t)v_1(t)}{\gamma-1} \frac{m_f}{\xi_f} \int_{\xi=0}^{\xi_f} \tilde{P}(\xi)\tilde{V}(\xi)d\xi. \end{aligned}$$

The expression for the kinetic energy is

$$\begin{aligned} E_K(t) &= \int_{m=0}^{m_f(t)} \frac{u^2(m,t)}{2} dm \\ &= \frac{u_1^2(t)}{2} \frac{m_f}{\xi_f} \int_{\xi=0}^{\xi_f} \tilde{U}^2(\xi)d\xi. \end{aligned}$$

It is easy to see that

$$\begin{aligned} p_1(t)v_1(t) &= W_v^{*\omega_p + \omega_v} a^{\alpha_p + \alpha_v} t^{\tau_p + \tau_v}, \\ u_1^2(t) &= W_v^{*2\omega_u} a^{2\alpha_u} t^{2\tau_u}. \end{aligned}$$

The dimensions of the product $p_1(t)v_1(t)$ and $u_1^2(t)$ are equal:

$$[p_1(t)v_1(t)] = [u_1^2(t)] = [\text{energy/mass}] = L^2\theta^{-2}.$$

Because only one combination of W_v^* , a and t with these dimensions exists, it follows that

$$p_1(t)v_1(t) = u_1^2(t).$$

The relation between the kinetic and thermal energies at time t is

$$\frac{E_T(t)}{E_K(t)} = \frac{p_1(t)v_1(t)}{u_1^2(t)} \frac{2}{\gamma-1} \frac{\int_0^{\xi_f} \tilde{P}(\xi)\tilde{V}(\xi)d\xi}{\int_0^{\xi_f} \tilde{U}^2(\xi)d\xi} = \frac{2}{\gamma-1} \eta(\xi_f),$$

where $\eta(\xi_f)$ is a numerical constant independent of time. We can define $E_1(t)$ as

$$E_1(t) = u_1^2(t) m_f / \xi_f = p_1(t) v_1(t) m_f / \xi_f,$$

and the total energy at time t takes the form

$$\begin{aligned} E_{\text{tot}}(t) &= E_1(t) \left[\int_0^{\xi_f} \left(\frac{\tilde{P}(\xi) \tilde{V}(\xi)}{\gamma - 1} + \frac{\tilde{U}^2(\xi)}{2} \right) d\xi \right] \\ &= E_1(t) \tilde{E}(\xi_f), \end{aligned}$$

where $\tilde{E}(\xi_f)$ is a numerical constant.

Appendix 2: Evaluation of the constants for radiation heat conduction in a multiply ionized material.

The numerical results obtained in this thesis were calculated with certain values for the constants and exponents appearing in eqs. (4-6). These values were chosen in an attempt to model approximately the heating of a dense, high-Z material (solid gold) by radiation heat conduction. In this appendix we outline the modelling which led us to these values. The heated material is considered as a partially ionized plasma, composed of free ions (with mean charge Z and mass m_i) and electrons; the ionization energy is neglected. The plasma is assumed to be neutral with $N_e = ZN_i$, where N_e and N_i are the number densities of electrons and ions, respectively. The plasma pressure is given by

$$p = N_i k(ZT + T_i) = NkT(Z+T_i/T)$$

where T and T_i are the electron and ion temperatures, and k the Boltzmann constant. We allow a temperature-dependent ionization according to

$$(Z+T_i/T) = Z_0 T^\vartheta$$

where Z_0 and ϑ are constants. From this it follows that

$$pv = KT^\delta$$

where $K = kZ_0/m_i$ and $\delta = \vartheta + 1$. Electrons and ions have f degrees of freedom. The internal energy per unit mass is then given by

$$e = fk(ZT+T_i)/2m_i = pv/(\gamma-1)$$

where $\gamma = (f+2)/f$. As numerical values we have assumed $K = kZ_0/m_i = 7.4 \times 10^3 \text{ cm}^2 \text{ s}^{-2} \text{ K}^{-3/2}$ and $\delta = \vartheta+1 = 3/2$ for gold ($m_i = 3.27 \times 10^{-22} \text{ g}$).

This corresponds to the relation

$$(Z+T_i/T) = 60 \sqrt{T(\text{keV})}$$

For γ we assumed $\gamma = 5/4$.

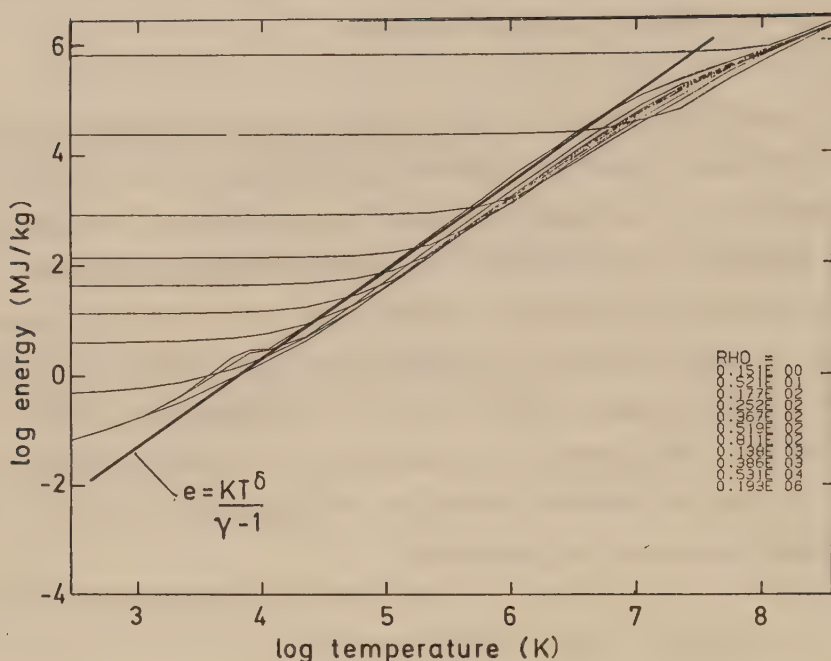


Fig. A1: Equation of state for gold at different densities ρ after the Sesame library. It is also shown the curve $e = KT^\delta/(\gamma-1)$ with $K = 7400 \text{ erg/g K}^{3/2}$, $\delta = 3/2$ and $\gamma = 5/4$.

On Fig. A1 we show the equation of state calculated above together with the curves taken from the SESAME Library (which became available during the course of this work). We can see that in the range $T \cong 10^6 \text{ K}$, $\rho \cong 1 \text{ g/cm}^3$ the agreement is very good.

For radiation heat conduction the expression for the heat flux is given by

$$S = -(16/3)\sigma T^3 \ell_R \partial T / \partial x$$

where σ is the Stefan-Boltzmann constant.

The Rosseland radiation mean free path ℓ_R due to bound-free and free-free absorption is of the form (see eq. 5.55 in Ref. 2)

$$\ell_R = A_\ell T^j \nu^{\mu'}$$

For the multiply ionized region we choose $j = 2$ and $\mu' = 2$. For gold ($\rho_s = 19.3 \text{ gcm}^{-3}$) one calculates $A_\ell = 8.6 \times 10^{-16} \text{ g}^2 \text{ cm}^{-5} \text{ K}^{-2}$. By comparing $S = -\kappa \partial T / \partial x$ with $\kappa = \kappa_o T^v \nu^{\mu'}$ one obtains

$$\kappa = (16/3) \sigma A_\ell T^{j+3} \nu^{\mu'}$$

Hence in our case we have $v = j+3 = 5$, $\mu' = \mu = 2$, and $\kappa_o = (16/3) \sigma A_\ell$. For the constants a and λ it follows that $a = \kappa_o / \delta K^{(v+1)/\delta} = 5.7 \times 10^{-35} \text{ s}^5 \text{ g}^3 \text{ cm}^{-13}$ and $\lambda = (v+1-\delta)/\delta = 3$.

If ℓ_R is written in the form $\ell_R = \ell_o w^s \nu^r$ one obtains $\ell_o = A_\ell / K^{j/\delta} = 5.96 \times 10^{-21} \text{ g}^2 \text{ s}^{8/3} \text{ cm}^{-23/3}$, $s = 4/3$ and $r = 2$.

The relations obtained above loose their validity for temperatures such that $Z_o T^\delta >$ atomic number i.e. when the material is fully ionized. For a fully ionized gas with atomic number Z one has

$$p = k(Z + T_i/T) N_i T$$

Comparing with the expression $p\nu = KT^\delta$ one finds that $K = kZ/m_i$ and $\delta = 1$. From eqs. (5.23) and (5.24) of Ref. 2 we find:

$$\ell_R = 1.55 \times 10^{80} (m_i/Z)^{11/2} Z^{-1} w^{7/2} \nu^2$$

$$\ell_p = 4.94 \times 10^{78} (m_i/Z)^{11/2} Z^{-1} w^{7/2} \nu^2$$

which gives $\ell_o = 4.94 \times 10^{78} (m_i/Z)^{11/2} Z^{-1}$; $s = 7/2$; $r = 2$

and $a = (5.60 \times 10^{14} m_i/Z)^{19/2} Z^{-1}$; $\lambda = 13/2$; $\mu = 2$

Appendix 3: Evaluation of the spectral energy density of the source from the measured spectra.

In this appendix we derive the relations by which the emitted spectral energy density of the source is calculated from the spectra obtained on film. We consider first the zeroth order which constitutes a pinhole image of the radiation source. The first order spectrum is then obtained by superposing such pinhole images, each one corresponding to the image generated by a different wavelength as shown in Fig. A.2.

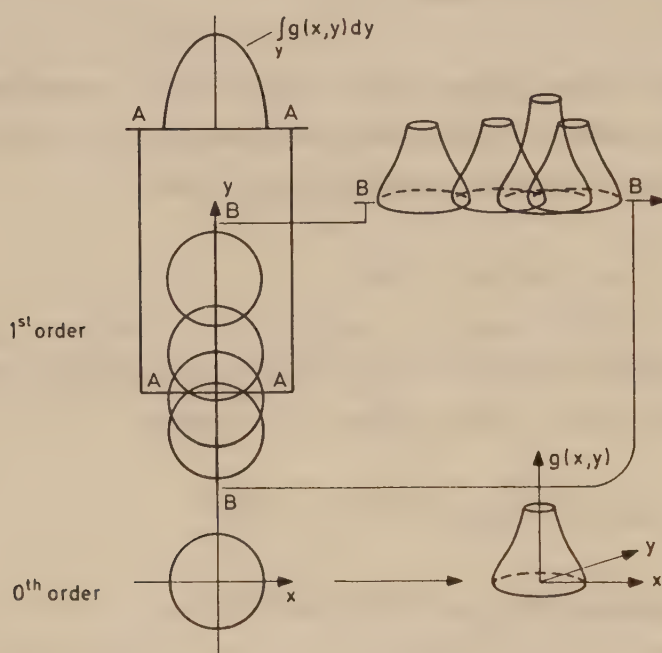


Fig. A2: Schematic representation of the spectrum. Shown is the zeroth order image (a pinhole picture of the radiation source) with a typical, rotationally symmetric profile $g(x, y)$. The first order spectrum is considered as the superposition of many zeroth order images, each one corresponding to a different wavelength as shown in the cut BB. The cut AA shows the profile perpendicular to the dispersion direction; this profile is to be interpreted as the integral over the zeroth order images along the y-direction: $\int_y g(x, y) dy$.

a) Zeroth order image

Under the assumption that the diagnostic hole radiates uniformly, the energy falling on a surface element of the zeroth order image of the source (diagnostic hole) is proportional to the portion of the source area

seen from this surface element. If the element with area $dxdy$ is centered at a point with coordinates (x,y) in the image plane, the energy falling on it is given by

$$W_o(x,y)dxdy = Bqg(x,y)tf_o dxdy/R^2 \quad (A1)$$

where:

- $W_o(x,y)$ is the measured energy density at the point (x,y) .
 $dxdy/R^2 = d\Omega$ is the differential solid angle of the surface element $dxdy$ as seen from the center of the radiation source (see Fig. A 3a).
 B is the radiant emittance of the source per unit solid angle (in the pinhole direction), per unit surface element and per unit time.
 q is the maximal source area one can see from the image plane.
 $g(x,y)$ is the dimensionless fraction of this area seen from a point (x,y) .
 t is the emission time.
 f_o is the grating efficiency (fractional transmission) in the zeroth order.

B can be written as

$$B = \int_{\lambda} I_{\lambda} d\lambda$$

where I_{λ} is the spectral energy density emitted by a unit surface element of the source in a unit time interval and per unit angle around the pinhole direction.

With respect to the directional dependence of the emitted radiation, we assume a $\cos\theta$ dependence, where θ is the azimuthal angle (in analogy with a black-body emitter). In this case the total radiant emittance into the half space is related to the energy B_o emitted per solid angle perpendicular to the source (i.e. into the direction of the grating) by:

$$\int_{\text{halfspace}} B(\Omega)d\Omega = \pi B_o$$

and similarly

$$\int_{\text{halfspace}} I_{\lambda}(\Omega)d\Omega = \pi I_{\lambda}(\theta=0)$$

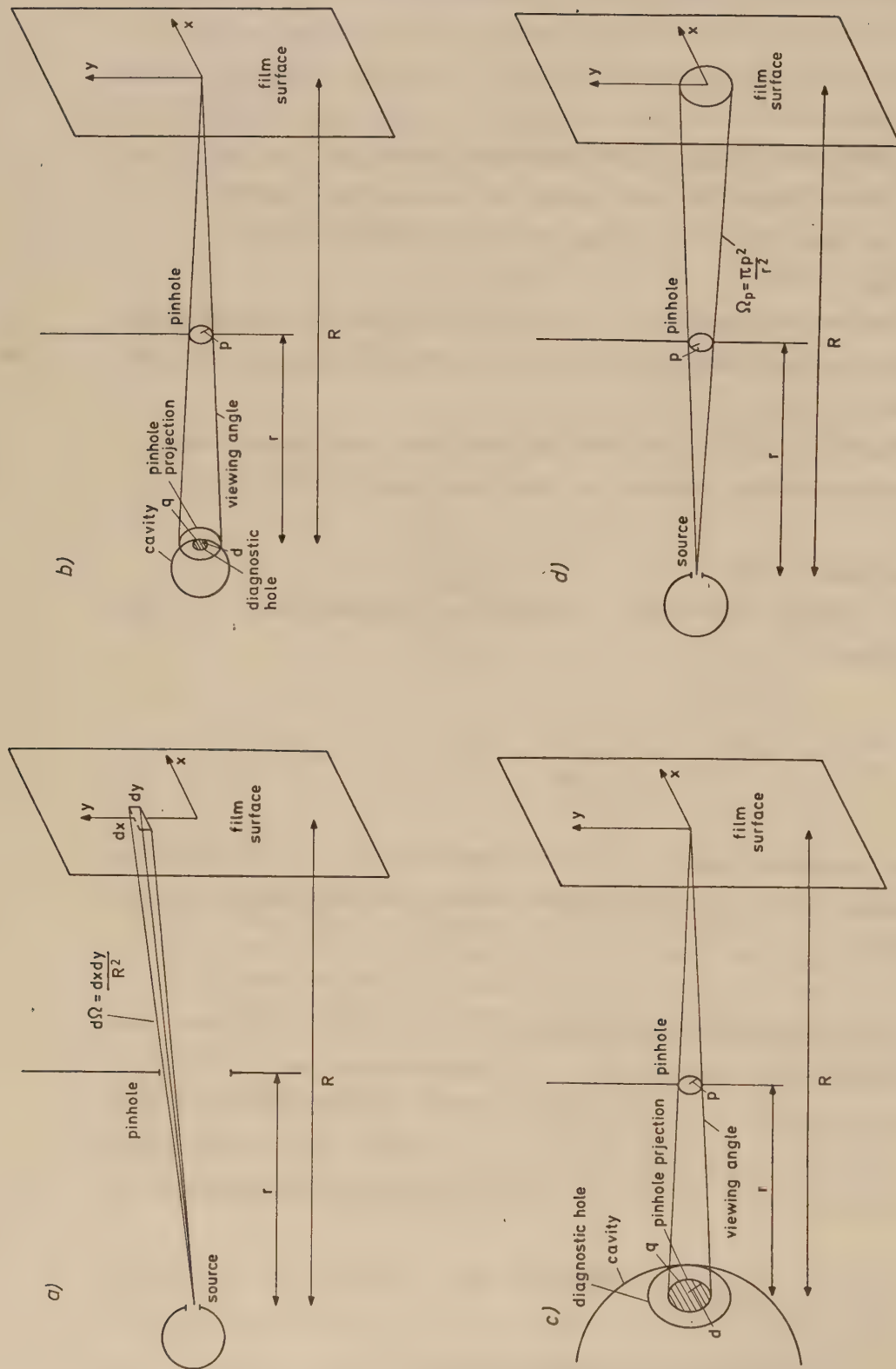


Fig. A3: Geometrical analysis of the zeroth order image.
a) the solid angle $d\Omega = dx dy / R^2$ of the surface element $dx dy$ as seen from the center of the radiation source.
b) maximal source area q visible from the image plane when the projection of the pinhole onto the source plane is larger than the diagnostic hole itself. In this case $q = \pi d^2$ ($d \equiv$ diagnostic hole radius).
c) maximal source area q visible from the image plane when the projection of the pinhole onto the source plane is smaller than the diagnostic hole itself. In this case $q = \pi p^2 R^2 / (R - r)^2$ ($p \equiv$ pinhole radius).
d) solid angle Ω_p of the pinhole as seen from the source. $\Omega_p = \pi p^2 / r^2$.

The maximal source area q can take on two values, depending on whether the pinhole projection from the film onto the source covers the entire diagnostic hole or not (see Fig. A3b and A3c). If $d \leq pR/(R-r)$ where d is the diagnostic hole radius and p the pinhole radius, the projection of the pinhole onto the source plane is larger than the diagnostic hole itself. In this case we have $q = \pi d^2$, i.e. q is equal to the area of the diagnostic hole. If $d > pR/(R-r)$, the projection of the pinhole onto the source plane is smaller than the diagnostic hole itself and $q = \pi p^2 R^2 / (R-r)^2$, i.e. q is equal to the area enclosed by the pinhole projection.

The total energy arriving at the zeroth-order image on the film plane is given by the surface integral:

$$\int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} W_0 dx dy = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} (Bqg(x,y)tf_0/R^2) dx dy$$

On the other hand this integral is equal to

$$\int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} W_0 dx dy = B\pi d^2 \Omega_p tf_0 \quad (A2)$$

where πd^2 is the entire source area and $\Omega_p = \pi p^2 / r^2$ is the solid angle of the pinhole as seen from the source (Fig. A3d). Note that this relation still holds in the presence of diffraction at the pinhole (as long as the dimension of the pinhole is large compared with the wavelength).

Fig.A4 illustrates the definition of the quantities p' and Q' ; $p' = pR/(R-r)$ is the projection of the pinhole radius onto the source plane; the distance between the diagnostic hole center and the center of the pinhole projection is given by $Q' = Qr/(R-r)$ where Q is the distance between the center of the zeroth-order image and the observation point. The fraction of the diagnostic hole seen by the point Q is the common area shared by the diagnostic hole and the pinhole projection. We now calculate this fraction as a function of the distance between the center of the zeroth order image ($x=0, y=0$) and the observation point. Fig. A5 shows p' and d together with Q' in the source plane. The origin of coordinates is

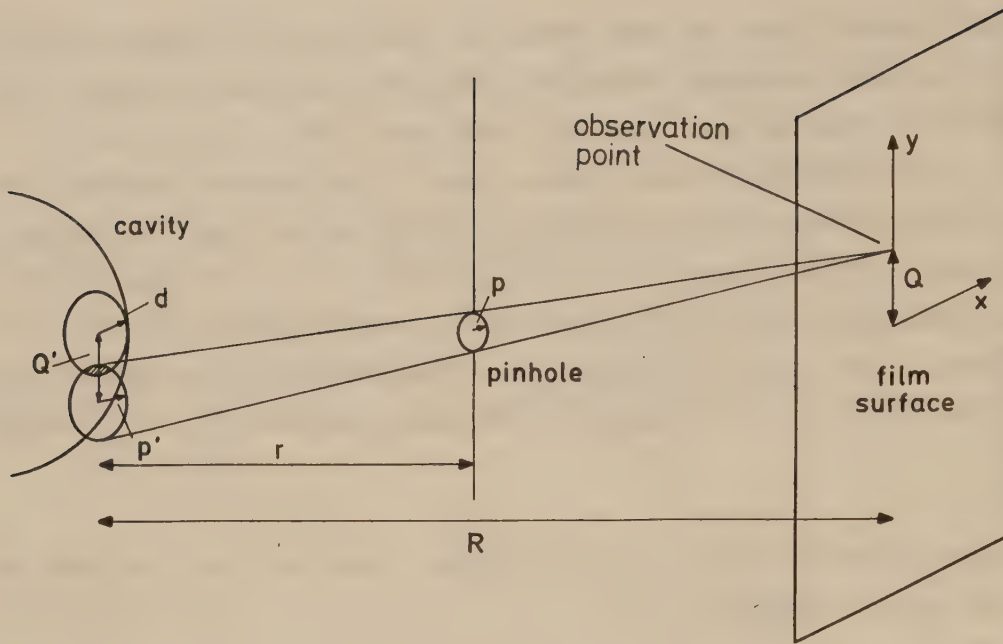


Fig. A4: Shown are, for a given observation point (x, y) , the distance Q to the origin of coordinates in the film plane, and, in the source plane, the distance Q' between the center of the diagnostic hole and the pinhole projection; p' is the projection of the pinhole radius p onto the source plane. The dashed area corresponds to the source area seen from the observation point (x, y) .

assumed to coincide with the center of the pinhole projection. Due to the rotational symmetry of the problem we can locate the center of the diagnostic hole on the x -axis without loss of generality. It is seen in Fig. A5 that both circles intersect on two points with coordinates (x_0, y_0) and $(x_0, -y_0)$. It is easy to prove that

$$x_0 = (p'^2 - d^2 + Q'^2) / 2Q' \text{ and } y_0 = (p'^2 - x_0^2)^{1/2}$$

Let $g(Q)$ be the area common to both circles. As discussed before there exist two cases; each case can be treated in the source plane or alternatively in the film plane. These are:

	in the source plane	in the film plane
I	$p' \geq d$	$p \geq d(1-r/R)$
II	$p' < d$	$p < d(1-r/R)$

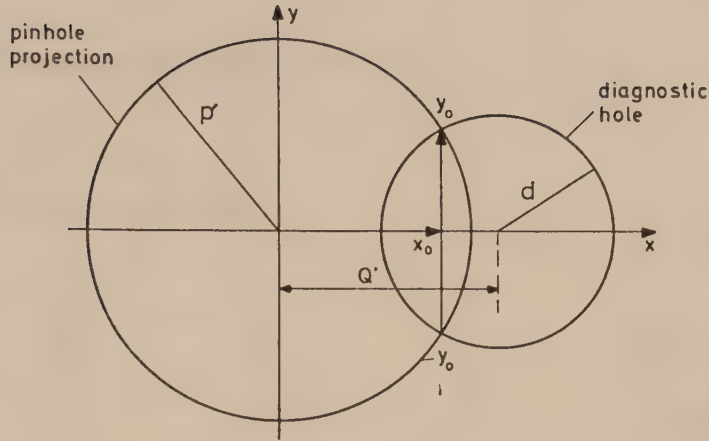


Fig. A5: Diagnostic hole and pinhole projection onto the source plane.
The center of both circles are separated by a distance Q' and both circles intersect at the points (x_0, y_0) and $(x_0, -y_0)$.

For each case exist four different possibilities:

on the source plane

on the film plane

1) $|p' - d| \geq Q'$

$|pR/r - d(R-r)/r| \geq Q$

2) $|p' - d| < Q' \leq (|p'^2 - d^2|)^{1/2}$

$|pR/r - d(R-r)/r| < Q \leq (|(pR/r)^2 - (d(R-r)/r)^2|)^{1/2}$

3) $(|p'^2 - d^2|)^{1/2} < Q' \leq p' + d$

$(|(pR/r)^2 - (d(R-r)/r)^2|)^{1/2} < Q \leq pR/r + d(R-r)/r$

4) $p' + d < Q'$

$pR/r + d(R-r)/r < Q$

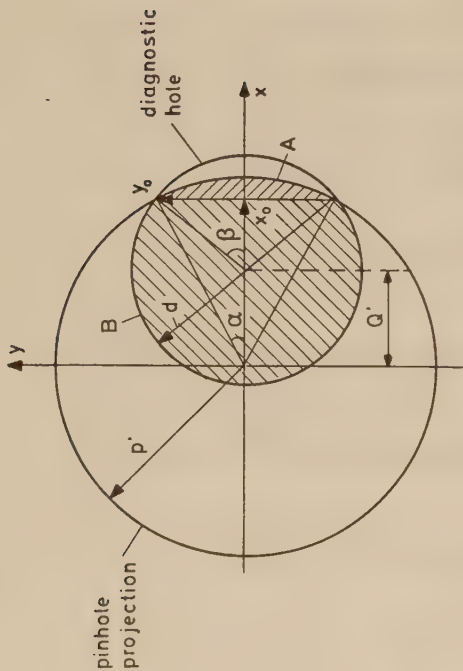


Fig. A6: Common area to both circles in the case $p' \geq d$ and $p'-d < Q'$.

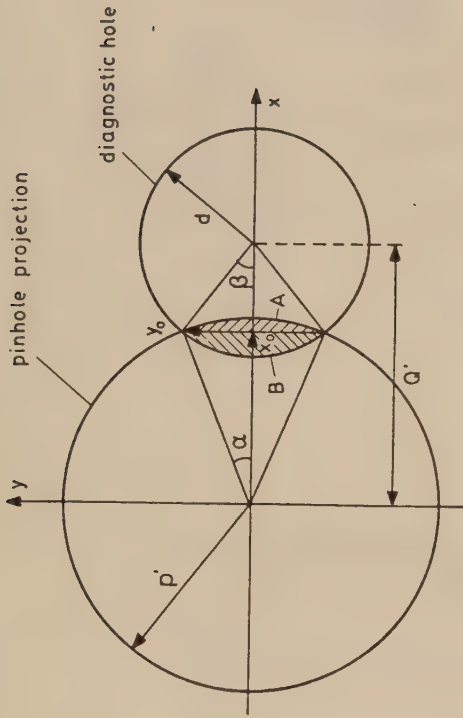


Fig. A7: Common area for both circles in the case $p' \geq d$ and $(1/2 - d^2/1)^{1/2} < Q' \leq p'+d$.

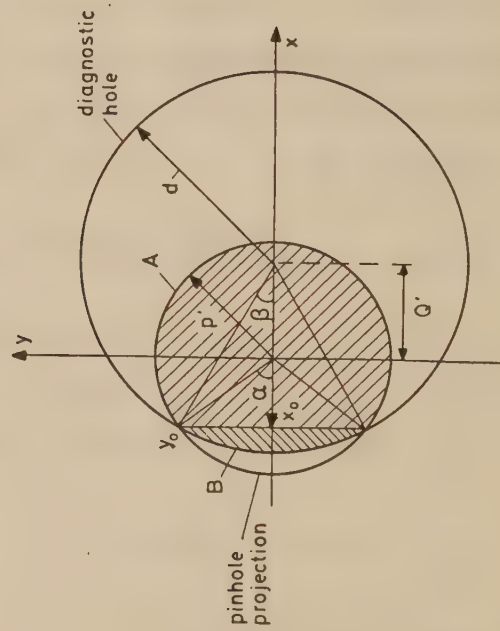


Fig. A8: Common area to both circles in the case $p' < d$ and $p'-d < Q'$.

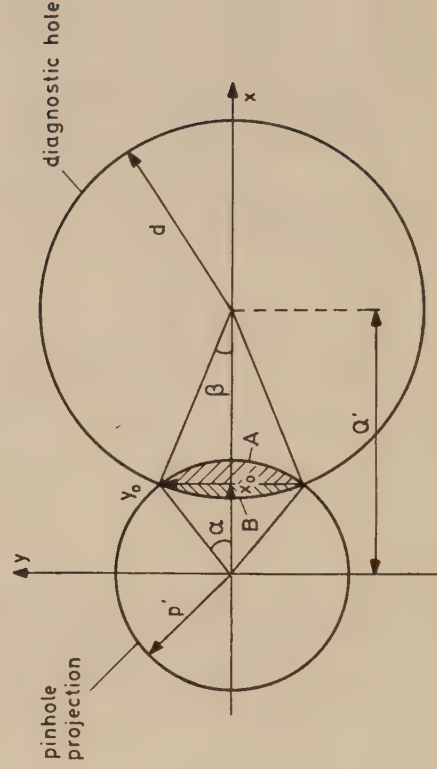


Fig. A9: Common area to both circles in the case $p' < d$ and $(1/2 - d^2/1)^{1/2} < Q' \leq p'+d$.

One can see here, that the width of the zeroth-order image is given by $pR/r + d(R-r)/r$ which can be directly interpreted as the addition of the pinhole and diagnostic hole projections into the film plane.

We proceed now to calculate $g(Q)$ for the eight cases.

I.1) $g = 1$

I.2) From Fig. A6 we see that

$$A = p'^2 \alpha - x_o y_o$$

$$B = \pi d^2 - \beta d^2 + y_o (x_o - Q')$$

where

$$\alpha = \arctg (y_o/x_o), \beta = \arctg (y_o/(x_o - Q')),$$

$$g = (A+B)/\pi d^2 = (p'^2/\pi d^2) \arctg (y_o/x_o) - (y_o Q'/\pi d^2) + \\ + 1 - (1/\pi) \arctg (y_o/(x_o - Q'))$$

I.3) From Fig. A7 we see that

$$A = p'^2 \alpha - x_o y_o$$

$$B = d^2 \beta - y_o (Q' - x_o)$$

where $\alpha = \arctg (y_o/x_o), \beta = \arctg (y_o/(Q' - x_o)),$

$$g = (A+B)/\pi d^2 = (p'^2/\pi d^2) \arctg (y_o/x_o) - (y_o Q'/\pi d^2) + \\ + (1/\pi) \arctg (y_o/(Q' - x_o))$$

I.4) $g = 0$

II.1) $g = 1$

II.2) From Fig. A8 we see that

$$A = \pi p'^2 - \alpha p'^2 + x_o y_o$$

$$B = \beta d^2 - y_o (x_o + Q')$$

where $\alpha = \arctg (y_0/x_0)$, $\beta = \arctg (y_0/(x_0+Q'))$,

$$g = (A+B)/\pi p'^2 = (d^2/\pi p'^2) \arctg (y_0/(Q'+x_0)) - \\ - (y_0 Q'/\pi p'^2) + 1 - (1/\pi) \arctg (y_0/x_0)$$

II.3) From Fig. A9 we see that

$$A = p'^2 \alpha - x_0 y_0$$

$$B = d^2 \beta - (Q' - x_0) y_0$$

where $\alpha = \arctg (y_0/x_0)$, $\beta = \arctg (y_0/(Q' - x_0))$,

$$g = (A+B)/\pi p'^2 = (d^2/\pi p'^2) \arctg (y_0/(Q' - x_0)) - \\ - (y_0 Q'/\pi p'^2) + (1/\pi) \arctg (y_0/x_0)$$

II.4) $g = 0$

In Fig. A10 a-c we show plots of $g(Q) = g(Q/(pR/r + d(R-r)/r))$ for different pinholes and diagnostic holes.

b) First-order spectrum

We proceed now to analyze the first order spectrum.

Let the origin of coordinates be at the center of the zeroth-order image, the y-axis parallel to the spectrum and the x-axis perpendicular to it (see Fig. 15 and Fig. A2).

As we have emphasized at the beginning of this appendix, the first-order spectrum is formed as a superposition of zero order images. To be more precise, at a point (x, y) belonging to the first order spectrum, one has an energy density given by:

$$W_1(x, y) = \int_{y'=-\infty}^{\infty} E_{\lambda'} g(x, y-y') dy' \quad (A3)$$

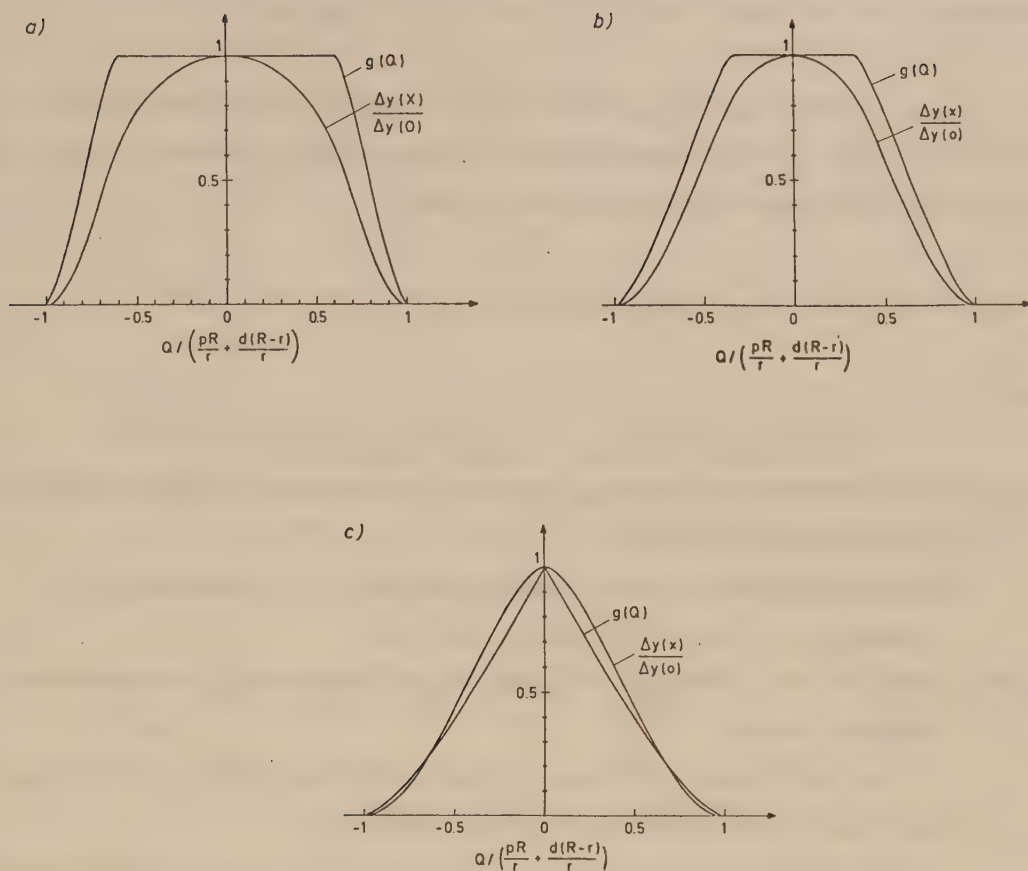


Fig. A10: Graphic representation of the profiles $g(Q)$ and $(\Delta y(x)/\Delta y(0))$ for the experimental parameters. $g(Q)$ is the profile of the zeroth-order image and $(\Delta y(x)/\Delta y(0))$ the normalized profile of a cut perpendicular to the first-order spectrum. The horizontal axis corresponds to the normalized distances $Q/(pR/r + d(R-r)/r)$ and $x/(pR/r + d(R-r)/r)$. The values of $\Delta y(0)$ are listed in Table A1.

a) Diagnostic hole radius $d = 100 \mu\text{m}$, pinhole radius $p = 12.5 \mu\text{m}$.

b) Diagnostic hole radius $d = 25 \mu\text{m}$, pinhole radius $p = 25 \mu\text{m}$,
or diagnostic hole radius $d = 100 \mu\text{m}$, pinhole radius $p = 25 \mu\text{m}$.
Both combination produce the same profile.

c) Diagnostic hole radius $d = 25 \mu\text{m}$, pinhole radius $p = 12.5 \mu\text{m}$.

In all cases $r = 30 \text{ cm}$, $R = 60 \text{ cm}$.

Here $E_{\lambda'}$ means the spectral energy density of the radiation with wavelength λ' . The relation between λ' and y' is given by the grating formula (valid for small θ'):

$$m\lambda' = a \sin\theta' \cong ay'/(R-r)$$

$R-r$ is the film-grating distance and $g(x, y-y')$ is the zeroth-order profile centered on the point (x, y') , i.e. $Q = (x^2 + (y-y')^2)^{1/2}$.

In this way, in analogy to equation (A1) and taking into account (A3), we can write the energy equation for the radiation coming onto a point (x, y) of the first-order spectrum as follows:

$$W_1(x, y) dx dy = \int_{y'=-\infty}^{\infty} I_{\lambda'} q g(x, y-y') t f_1 d\lambda dx dy' / R^2 \quad (A4)$$

where

$W_1(x, y)$ is the measured energy density on the first order spectrum around the point (x, y) .

$I_{\lambda'}$ is the emitted spectral energy density at a wavelength λ' related to y' by the grating formula.

$dx dy' / R^2 = d\Omega'$ is the differential solid angle formed by the surface element $dx dy'$.

f_1 is the diffraction efficiency into the first order, which we take as independent of the wavelength.

$d\lambda$ is the wavelength interval related to dy through the grating formula.

We now make the assumption that $I_{\lambda'}$ does not change strongly over the interval where $g(x, y-y')$ is different from zero (as is appropriate for our broadband, blackbody like spectra). In this case we can take $I_{\lambda'} \cong I_{\lambda}$ out from the integral and get:

$$W_1(x, y) dx dy = q t f_1 I_{\lambda} d\lambda \Delta y(x) dx / R^2$$

where we have defined

$$\Delta y(x) = \int_{y'=-\infty}^{\infty} g(x, y') dy'$$

$\Delta y(x)$ represents the profile of the energy density on the film perpendicular to the dispersion direction. In Fig. A10 a-c $\Delta y(x)$ is plotted in a normalized form as $\Delta y(x / (pR/r + d(R-r)/r)) / \Delta y(0)$ for the cases occurring in the experiment. The values of $\Delta y(0)$ are given in table A1. Note that $\Delta y(0)$ is of order $pR/r + d(R-r)/r$.

Table AI: Values of $\Delta y(0)/(pR/r+d(R-r)/r)$ as a function of the pinhole radius p and diagnostic hole radius d . For all cases $R = 60$ cm and $r = 30$ cm.

	p (μm)	
	12.5	25
$d(\mu\text{m})$		
25	1.0	1.3
100	1.6	1.3

Table AII: Wavelength below which the spectrometer resolution is determined by geometry rather than diffraction as a function of the pinhole radius p and diagnostic hole radius d . For all cases $R = 60$ cm, $r = 30$ cm.

	p (μm)	
	12.5	25
$d(\mu\text{m})$		
25	.83 Å	250 Å
100	208 Å	500 Å

At the center of the spectrum (that is $x = 0$) we get for the maximum energy density $W_{1\text{max}}(y) = W_1(x=0, y)$ the following expression:

$$W_{1\text{max}}(y)dx dy = q t f_1 I_\lambda d\lambda \Delta y(0) dx/R^2$$

From this equation we can obtain the emitted energy density I_λ as a function of the measured maximal energy density $W_{1\text{max}}(y)$:

$$I_\lambda = \frac{W_{1\text{max}}(y) R^2}{q (d\lambda/dy) t f_1 \Delta y(0)} \quad (\text{A5})$$

The wavelength resolution corresponding to the width of the zeroth-order image is given by:

$$\Delta\lambda = (a/(R-r))(pR/r+d(R-r)/r) = (a/r)(pR/(R-r)+d)$$

On the other hand, for a point source the wavelength resolution due to diffraction is given by

$$\Delta\lambda = \lambda/Nm = a\lambda/pm$$

Comparison shows that for $\lambda \leq (p/r)(pR/(R-r)+d)$ diffraction effects are negligible in front of the geometrical resolution. The corresponding wavelength is shown in Table AII for the experimental cases.

Eq. (A5) allows us to obtain I_λ as a function of $W_{1\max}(y)$, but it is sometimes more convenient or easier to measure the integral $\int_{x=-\infty}^{\infty} W_1(x,y)dx$ rather than $W_{1\max}(y)$. For this reason we will derive an equation connecting I_λ and $\int_{x=-\infty}^{\infty} W_1(x,y)dx$. After integration of equation A5 we obtain:

$$dy \int_{x=-\infty}^{\infty} W_1(x,y)dx = \int_{x=-\infty}^{\infty} \int_{y'=-\infty}^{\infty} I_{\lambda'} q g(x,y-y') t f_1 d\lambda' dx dy'/R^2 \quad (A6)$$

With the approximation that $I_{\lambda'} \cong I_\lambda$ where $g(x,y-y')$ is different from zero, we get for the right hand side of eq. A6:

$$q I_\lambda t f_1 d\lambda \int_{x=-\infty}^{\infty} \int_{y'=-\infty}^{\infty} g(x,y') dx dy'/R^2 \quad (A7)$$

In analogy with the derivation of eq. A2 we can write (A7) as:

$$I_\lambda t f_1 \pi d^2 \Omega_p d\lambda$$

and obtain the desired expression for I_λ

$$I_\lambda = \frac{\int_{x=-\infty}^{\infty} W_1(x,y)dx}{t f_1 (d\lambda/dy) \pi d^2 \Omega_p} \quad (A8)$$

Note that it is independent of the form of $g(x,y)$. Eq. (A5) was used for obtaining I_λ once $W_{1\max}(y)$ was measured for the big cavities and eq. (A8) for obtaining I_λ as a function of the measured value of $\int_{x=-\infty}^{\infty} W_1(x,y)dx$ for the small cavities.

The numerical values used in the calculation were $f_1 = 0.091$, $d\lambda/dy = a/(R-r) = 333.3 \text{ \AA/cm}$ and $t = 3 \times 10^{-10} \text{ s}$ with $a = 10^{-4} \text{ cm}$, $R = 60 \text{ cm}$, $r = 30 \text{ cm}$. One obtains $\Delta\Omega_p = 5.45 \times 10^{-9}$ for small pinholes and $\Delta\Omega_p = 1.89 \times 10^{-9}$ for big pinholes.

Appendix 4: Photon range in heated gold

We calculate the absorption coefficient for heated gold from eq. 5.44 of Ref. /2/:

$$\kappa_V = 0.96 \times 10^{-7} N z^2 x_1 T^{-2} x^{-3} \text{ cm}^{-1} \quad (\text{for } h\nu > I)$$

where: N: ion density; z: mean ionization degree; T: matter temperature in degree Kelvin; $x = h\nu/kT$; $h\nu$: photon energy; k: Boltzmann constant; $x_1 = I/kT$; I: ionization energy.

The mean ionization degree is calculated from $z \cong 60\sqrt{t(\text{keV})}$ (see App. 2). For a temperature of 100 eV the equation 3.56 of Ref. /2/ gives for gold of solid density $I \cong 2 \text{ kT}$. This yields the expression given in Table VIII.

References

- /1/ M. Planck, *Annalen der Physik* 4, 553 (1900)
- /2/ Ya. B. Zel'dovich and Yu. P. Raizer,
Physics of Shock Waves and High-Temperature Hydrodynamic
Phenomena, Academic Press, New York and London 1966.
- /3/ E. Buckingham, *Phys. Rev. (Ser. II)* 4, 345 (1914)
- /4/ L.I. Sedov, *Similarity and Dimensional Methods in Mechanics*,
Academic Press Inc., New York 1959.
- /5/ G.I. Barenblatt, *Similarity, Self-Similarity, and Intermediate
Asymptotics*, Consultants Bureau (a Division of Plenum Publishing
Corporation), New York 1979.
- /6/ J.L. Bobin, *Phys. Fluids* 14, 2341 (1971).
- /7/ K. Nozaki and K. Nishihara, *J. Phys. Soc. Jpn.* 48, 993 (1980).
- /8/ R.E. Marshak, *Phys. Fluids* 1, 24 (1958)
- /9/ S.I. Anisimov, *JETP. Lett.* 12, 287 (1970).
- /10/ A. Barrero and Juan R. Sanmartin, *Phys. Fluids* 20, 1155 (1977).
- /11/ Juan R. Sanmartin and A. Barrero, *Phys. Fluids* 21, 1957 (1978).
- /12/ Juan R. Sanmartin and A. Barrero, *Phys. Fluids* 21, 1967 (1978).
- /13/ Iu. V. Afanas'ev, V.M. Krol', O.N. Krokhin and I.V. Nemchinov,
Applied Mathematics and Mechanics 30, 1218 (1967).
- /14/ J. Bernstein and F.J. Dyson, General Atomic Report No. GA-848,
1959; see also B.H. Armstrong, R.R. Johnston, and P.S. Kelly,
J. Quant. Spectrosc. Radiat. Transfer 5, 55 (1965).

- /15/ I.V. Nemchinov, Applied Mathematics and Technical Physics 1, 36 (1960)
- /16/ J. Meyer-ter-Vehn, R. Pakula, R. Sigel and K. Unterseer, Opt. Commun. 104, 410 (1984).
- /17/ G. Brederlow, E. Fill, K.J. Witte, The High-power Iodine Laser (Springer-Verlag, Berlin Heidelberg, 1983).
- /18/ E. Fill, Opt. Commun. 33, 321 (1980).
- /19/ K. Eidmann, F. Amiranoff, R. Fedosejevs, A. Maaswinkel, R. Petsch, R. Sigel, G. Spindler, Yung-lu Teng, G. Tsakiris, S. Witkowski, Phys. Rev. A30, 2568 (1984).
- /20/ Peter Predehl, Thesis Dissertation, Friedrich-Alexander-Universität, Erlangen-Nürnberg, February 1980;
see also H. Bräuninger et al. Appl. Optics 18, 368 (1979);
ibid. p. 2906, p. 3502.
- /21/ Tetsuo Kishimoto, Thesis Dissertation, Ludwig-Maximilian-Universität, München (to be submitted in 1985).
- /22/ K.H. Hellwege, Einführung in die Physik der Atome,
Springer-Verlag Berlin Heidelberg, 1974.
- /23/ R.J. Harrach and R.E. Kidder, Phys. Rev. A23, 887 (1981).
- /24/ L.V. Spencer, National Bureau of Standards Monograph 1, 1959
- /25/ Braginskii, Reviews of Plasma Physics Vol. I (Consultants Bureau New York, 1965) p. 215.
- /26/ T. Mochizuki, T. Yabe, K. Okada, M. Hamada, N. Ikeda, S. Kiyokawa, C. Yamanaka, Report ILE 8411P (September 1984),
Institute of Laser Engineering, Osaka University, Osaka, Japan.

/27/ x-ray cross section compilation from 0.1 keV to 1 MeV. Discussion and results. Vol. 1 (Rev. 1). Wm. J. Veigele et al. Kaman Sciences Corporation, Colorado Springs, Colorado, 1971.

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